WELL-BALANCED SCHEME FOR A KINETIC-PARABOLIC SYSTEM MODELING TRAVELING CHEMOTACTIC AGGREGATES

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We present numerical approximation a model consisting in a kinetic equation for " run-and-tumble " biased motion of bacteria,

$$\partial_t f + v\partial_x f = \int_V \mathcal{T}(t,x,v')f(t,x,v')d\nu(v') - \mathcal{T}(t,x,v)f(t,x,v)$$

with tumbling rate $\mathcal{T}(t,x,v)$ depending on time, space and velocity through an aggregating chemical signal M and a nutrient N

$$\mathcal{T}(t,x,v') = 1 + \chi_M \cdot \phi\left(\left.\frac{DM}{Dt}\right|_{v'}\right) + \chi_N \cdot \phi\left(\left.\frac{DN}{Dt}\right|_{v'}\right), \qquad \phi(\cdot) = -\mathrm{sign}(\cdot),$$

where $\frac{D}{Dt}(\cdot)$ is a material derivative, coupled with two reaction-diffusion equations for chemoattractants

$$\begin{cases} \partial_t M = D_M \partial_{xx} M - \alpha M + \beta \int_V f(t,x,v')d\nu(v), \\ \partial_t N = D_N \partial_{xx} N - \gamma \int_V f(t,x,v')d\nu(v)N. \end{cases}$$

The model displays time-asymptotic propagation at constant velocity, i.e. aggregated traveling (exponential) layers. Their construction was investigated by V.Calvez (2016) and it was proved that traveling wave solutions exist under certain conditions on the parameters. Moreover such waves are not unique in general, and far from the diffusive regime, waves traveling at different velocities are proved to co-exist (bi-stability). We describe a well-balanced setup based on both " Case's elementary solutions " for the kinetic part and L-splines reconstruction for the parabolic system, which is able to capture the waves over long period of time, despite their lack of regularity. We present numerical results of local asymptotic stability of some cases where several traveling wave co-exist.