Weighted Hardy's inquality

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Abstract

We give general condition to establish the weighted Hardy's inequality

$$c\int_{\mathbb{R}^N}\frac{\varphi^2}{|x|^2}d\mu \le \int_{\mathbb{R}^N}|\nabla\varphi|^2 d\mu + C\int_{\mathbb{R}^N}\varphi^2 d\mu \quad \varphi\in C_c^\infty(\mathbb{R}^N),\ c\le C^*(N,\mu) \tag{1}$$

with respect the probability measure $d\mu$. Moreover the optimality of the C^* is given. The inequality is related to the following Kolmogorov equation perturbed by a singular potential

$$Lu = \Delta u + \frac{\nabla\mu}{\mu}\cdot\nabla u + \frac{c}{|x|^2}u \tag{2}$$

for which the existence of the solution to the corresponding parabolic problem can be proved. The hypotheses on $d\mu$ allow the drift term to be of type $\frac{\nabla\mu}{\mu} = -|x|^r x$ with $r\ge -2$ or also $(|x|^r \log|x|)\,x$.