Dynamical systems on networks: well-posedness, graph realizability and asymptotic state lumping

Part I. J. Banasiak[1,2], Diffusion processes

Part II. and A. Falkiewicz[2], Transport processes

[1] School of Mathematics, Statistics and Computer Science
University of KwaZulu-Natal, Durban, South Africa

[2] Institute of Mathematics, Łódź University of Technology, Łódź, Poland

e-mail: banasiak@ukzn.ac.za

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ABSTRACT

Recently there has been an interest in dynamical problems on graphs, where some evolution operators, such as transport or diffusion, act on the edges of a graph and interact through nodes. One can mention here quantum graphs, see e.g. [5, 8], diffusion on graphs in probabilistic context, [2, 7], transport problems, both linear and nonlinear, [1, 4, 3], migrations, [6], and several other applications discussed in e.g. [9, 10]. In this note we shall focus on a more general linear transport and diffusion problems posed on networks consisting of one dimensional domains, which are coupled through transmission conditions between an arbitrary selection of the endpoints of the domains. This allows for communication between domains which not necessarily are physically connected and makes it possible to consider within the same framework not only classical transport and diffusion problems on graphs but also models such as Rotenberg type models describing mutations in dividing cells, [11].

In these two talks we address the following problems:

1. Well-posedness of the problems in the sense of C_0-semigroup theory and positivity of resulting semigroups;
2. Conditions under which such a generalized model describes a classical model on a metric graph;
3. The so-called asymptotic state lumping; that is, conditions under which such network problems can be approximated by appropriately constructed system of ordinary differential equations.

The talk is delivered in two parts.

I. Diffusion processes;

II. Transport processes.

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