On singular twistings of Banach spaces

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Joint work with J. Castillo and M. González

Some questions about self-extensions

1) X =self-extension of ℓ_p and X has unc. basis $\Rightarrow X \simeq \ell_p$?
(Kalton 03: yes if $p = 2$)

2) Does any non-trivial twisted Hilbert space fail GL-l.u.st.?
(Johnson-Lindenstrauss-Schechtman 80: yes for Z_2
Casazza-Kalton 96: more general result)

$\Rightarrow$ New examples?

3) Study $\Omega(x) = x \log \frac{|x|}{\|x\|}$ on any X with unc. basis (or even Köthe space structure)

4) Define and study self-extensions without lattice structure.

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X	$X \oplus_{KP} X$	
ℓ_p	singular $0 < p < +\infty$	$1 < p$ Kalton-Peck 79 $p \leq 1$ Cabello-Sanchez, Castillo, Suarez 12
L_p	singular $2 \leq p < +\infty$ non-singular $0 < p < +\infty$	CS, C, S 12 Suarez 13

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Note: singular $\Rightarrow$ disj. singular,
singular $\Leftrightarrow$ disj. singular if unconditional basis.

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A condition for singularity without unconditionality

Proposition

Assume we have a complex interpolation scheme of two spaces X_0, X_1 with a common 1-monotone basis. Assume that for every block-subspace W of X_θ, there exist sequences $\varepsilon_n, \lambda_n, M_n$ such that for every n there is a finite successive sequence $n < y_1 < \cdots < y_n$ with $\|y_i\| \leq 1\ \forall i = 1, \ldots, n$ such that

(i) *The block sequence is ε_n-optimal, in the sense that $\left\|\sum_{i=1}^n y_i\right\| \geq \varepsilon_n A_{X_0}(n)^{1-\theta} A_{X_1}(n)^\theta$;*

(ii) *The block sequence $\{y_1, \ldots, y_n\}$ is λ_n-unconditional;*

(iii) *the space $[y_1, \ldots, y_n]$ is M_n-complemented in X_θ;*

and so that $\liminf_{n\to+\infty} \frac{\lambda_n^3 M_n}{\varepsilon_n \left|\log \frac{A_{X_0}(n)}{A_{X_1}(n)}\right|} = 0$. *Then Ω_θ is singular.*

Recall: $A_X(n) = \sup_{n<x_1<\cdots<x_n, x_i \in B_X} \|x_1 + \cdots + x_n\|_X$

The same condition for interpolation of families

Proposition

Assume we have a complex interpolation scheme of spaces $(X_z)_{z\in\delta S}$ with a common 1-monotone basis. Assume that for every block-subspace W of X_θ, there exist sequences $\varepsilon_n, \lambda_n, M_n$ such that for every n there is a finite successive sequence $n < y_1 < \cdots < y_n$ with $\|y_i\| \leq 1\ \forall i = 1, \ldots, n$ such that

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Use: $A_j(n) = \mathrm{esssup}_{t\in\mathbb{R}} A_{X_{j+it}}(n)$

Merci

THANK YOU!