Conférence "Equations aux dérivées partielles et semi-groupes"

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[1] The dinner will be held at the restaurant "Le Poker d'As"

Elliptic Problems in Smooth and Non Smooth Domains

Chérif Amrouche

Abstract: We are interested here in questions related to the regularity of solutions of elliptic problems

$$\operatorname{div}(A\nabla u) = f, \text{ in } \Omega,$$

with Dirichlet or Neumann boundary condition. For the last 20 years, lots of work has been concerned with questions when A is a matrix or a function and Ω is a Lipschitz domain. We give here some complements and we extend this study to obtain regularity results for domains having an adequate regularity.
Using the duality method, we will then revisit the work of Lions-Magenes [3], concerning the so-called very weak solutions, when the data are less regular. Thanks to the interpolation theory, it permits us to extend the classes of solutions and then to obtain new results of regularity.
This a joint work with Mohand Moussaoui and Huy Hoang Nguyen (see [1]).

References

[1] C. Amrouche, M. Moussaoui, H.H. Nguyen. Laplace equation in smooth or non smooth domains. Work in Progress.

[2] D. Jerison, C.E. Kenig. The Inhomogeneous Dirichlet Problem in Lipschitz Domains, *J. Funct. Anal. 130, (1995),* 161 – 219.

[3] J.L. Lions, E. Magenes. *Problèmes aux limites non-homogènes et applications*. Vol. 1, Dunod, Paris, (1969).

[4] J. Necas. *Direct methods in the theory of elliptic equations*. Springer Monographs in Mathematics. Springer, Heidelberg, (2012).

Positive semigroups: Kernels and irreducibility

Wolfgang Arendt

Abstract: Consider a semigroup T on a $L^2(\Omega)$, where Ω is a bounded open set in the euclidean space. We will investigate under which conditions T has a continuous kernel defined on the closure of $\Omega \times \Omega$. In fact, there is a very simple characterization which will be presented in the talk.
An important property for positive semigroups is irreducibility which has interesting consequences for the asymptotic behavior and also spectral properties of the generator and the semigroup.
Sometimes it is hard to verify irreducibility, in particular on spaces of continuous functions. We will show how the continuous kernel can help a lot and as corollary, we will obtain a strictly positive continuous principal eigenvector defined on the closure of $\Omega \times \Omega$.
These results can be applied to the Laplacian with boundary conditions, more general elliptic operators and also to the Dirichlet to Neumann operator.
The talk is based on joint work with Tom ter Elst (Auckland).

References

[1] W. Arendt, T. ter Elst: The Dirichlet-to-Neumann operator on $C(\partial\Omega)$. arXiv:1707.05556

Discrete fragmentation-coagulation models with growth and decay - recent results

Jacek Banasiak

Abstract: Fragmentation-coagulation model, introduced exactly 100 years ago by Marian Smoluchowski, describe a variety of phenomena, from polymerisation/depolimerization, aerosol formation, creation of planetesimals, through blood agglutination, animal groupings to phytoplankton aggregation, [4]. In the discrete case this is an infinite system of nonlinear, nonlocal equations with unbounded coefficients. In many cases, especially in applications in natural sciences, the fragmentation-coagulation processes are accompanied by other phenomena, such as growth or death of the modelled organisms.
Systematic mathematical analysis of such equations goes back to the work of John Ball and Jack Carr, [1]. Using weak compactness methods, they provided weak solutions to the system of coagulation-fragmentation equations. In this talk we follow another approach, focusing first on the fragmentation process. Using substochastic semigroup theory, [2], we prove the analyticity and compactness for a class of fragmentation semigroups. This result allows for providing a precise description of the long term behaviour of discrete fragmentation processes with growth and decay and to prove the existence of classical solutions to equations describing such processes combined with coagulation, see [3].

References

[1] J. M. Ball and J. Carr. Asymptotic behaviour of solutions to the Becker-Döring equations for arbitrary initial data. *Proc. Roy. Soc. Edinburgh Sect. A*, 108(1-2):109–116, 1988.

[2] J. Banasiak and L. Arlotti. *Perturbations of positive semigroups with applications*. Springer Monographs in Mathematics. Springer-Verlag London, Ltd., London, 2006.

[3] J. Banasiak. Global classical solutions of coagulation-fragmentation equations with unbounded coagulation rates. *Nonlinear Anal. Real World Appl.*, 13(1):91–105, 2012.

[4] F.P. da Costa. Mathematical aspects of coagulation-fragmentation equations. In *Mathematics of Energy and Climate Change*, pages 83–162. Springer, 2015.

Hyperbolic phenomena in control of Parabolic Equations

Assia Benabdallah

Abstract: One of the main goal in control theory is to drive the state of the system to a given configuration using a control that acts through a source term located inside the domain or on the boundary.
The reference works for the control of linear parabolic problems are due to H.O. Fattorini and D.L. Russell, [1], in the 70's for the one dimensional case and to A.V. Fursikov, O.Yu. Imanuvilov, [2], on one side and G. Lebeau, L. Robbiano, [3], on the other side both in the 90/s for the multi-dimensional case. They established null-controllability of heat equations with distributed or boundary controls at any time $T > 0$ and for any control domain.
The aim of this talk is to give an overview on recent results on the controllability of parabolic systems. Through simple examples, I will show that new phenomena appear as, for instance, minimal time of control, geometrical dependence on the location of the control. These phenomena are well-known in control of hyperbolic equations but unexpected in parabolic equations. I will connect them to a quantified resolvent condition and end the talk by several open problems

References

[1] H.O. Fattorini, D. L. Russell, Exact controllability theorems for linear parabolic equations in one space dimension, Arch. Rational Mech. Anal. 43 (1971), 272–292.

[2] A. Fursikov, O. Yu. Imanuvilov, "Controllability of Evolution Equations", Lecture Notes Series 34, Seoul National University, Research Institute of Mathematics, Global Analysis Research Center, Seoul, 1996.

[3] G. Lebeau, L. Robbiano, Contrôle exact de l'équation de la chaleur, Comm. Partial Differential Equations 20 (1995), no. 1-2, 335–356.

Diffusive asymptotics for a kinetic model of mixture with general cross-sections

Laurent Boudin

Formal hydrodynamic limits of kinetic models can be obtained following two kinds of methods : the perturbative and the moment ones. In this talk, I present joint works with Bérénice Grec, Vincent Pavan et Francesco Salvarani, where, using the moment method, we investigate the formal diffusion limit of a kinetic model of a non-reactive gaseous mixture of Boltzmann type. In particular, this allows to state the integral expressions of the diffusion coefficients arising in the Maxwell-Stefan equations with respect to general cross-sections.

A quantitative Faber-Krahn inequality for the Robin Laplacian eigenvalue

Dorin Bucur

Abstract: The Faber-Krahn inequality for the first Robin Laplacian eigenvalue was proved by Bossel 1986 in two dimensions of the space and by Daners 2005 in any dimension of the space, in the context of Lipschitz sets. In this talk, I give a quantitative form of this Faber-Krahn inequality, which moreover applies to arbitrary open sets. The asymmetry term involves the square power of the Fraenkel asymmetry, multiplied by a constant depending on the Robin parameter, the dimension of the space and the measure of the set. The proof is based on the analysis of qualitative properties of a family of free discontinuity problems. This is a joint work with E. Ferone, C. Nitsch and C. Trombetti.

Improved duality estimates and applications

Laurent Desvillettes

Abstract: Improved duality estimates enable to show that solutions to a special type of singular parabolic equations lie in L^p for some $p > 2$. Thanks to this estimate, it is possible to improve results in various problems arising in reaction-diffusion systems, reaction-cross-diffusion systems and coagulation-fragmentation-diffusion systems.

On Koopman semigroups on L^p-spaces

Nikolai Edeko

Abstract: In the theory of strongly continuous operator semigroups, a classical result by Derndinger and Nagel characterizes those semigroups on $C(K)$-spaces consisting of lattice homomorphisms, showing that they are precisely the Koopman semigroups corresponding to continuous semiflows on K. Motivated by recent similar results for unitary groups on L^2-spaces by ter Elst and Lemanczyk, we discuss how a similar characterization can be obtained on L^p-spaces.

On the domain of a Schrödinger operator with complex potential – Old and New

Bernard Helffer

Abstract: The aim of this talk is to review and compare the spectral properties of (the closed extension of) $-\Delta+U$ ($U \geq 0$) and
$-\Delta + iV$ in $L^2\left(\mathbb{R}^d\right)$ for C^∞ potentials U or V with polynomial behavior.
The case with magnetic field is also considered. More precisely, the aim is to compare the criteria for:

- Essential selfadjointness (esa) or maximal accretivity (maxacc),
- Compactness of the resolvent,
- Maximal inequalities,

for these operators.
The most recent results devoted to the Schrödinger operator with complex potential have been obtained in collaboration with Y. Almog (2016) and J. Nourrigat (2017).

Le point sur la minimisation de λ_k

Antoine Henrot

Abstract: Désignons par $\lambda_k(\Omega)$ la k-ième valeur propre du Laplacien avec condition de Dirichlet sur l'ouvert borné Ω. Un problème d'optimisation de forme classique consiste à chercher à minimiser $\lambda_k(\Omega)$ sous contrainte de volume. Les références [1] et [2] fournissent une introduction assez complète au sujet.
Ces dernières années, la contrainte de périmètre a également été étudiée. Nous commencerons par rappeler l'état de l'art pour ces deux problèmes. Nous donnerons ce qui nous parait être quelques problèmes ouverts intéressants. Puis nous envisagerons le cas d'une contrainte sur le diamètre de Ω pour lequel nous donnerons quelques résultats très récents.
[1] A. Henrot. Extremum problems for eigenvalues of elliptic operators, Birkhäuser Verlag 2006.
[2] A. Henrot (ed.) Shape optimization and spectral theory. De Gruyter (open access) 2017.

Discrete coagulation-fragmentation systems

Lyndsay Kerr

Abstract: In many situations in nature and industrial processes clusters of particles can combine into larger clusters or fragment into smaller clusters. The evolution of these particles can be described by differential equations known as coagulation-fragmentation equations. In the discrete size case it is assumed that the mass of each cluster is a natural number and a cluster of mass n consists of n identical units. The main part of the talk will concentrate on the case of pure discrete fragmentation. Here, the theory of substochastic C_0-semigroups can be used to obtain results relating to the existence of a unique, positive, mass conserving solution. The full coagulation-fragmentation system, where the coagulation coefficients may be time-dependent, will also be briefly examined.

On the fixed space of a Koopman operator

Kari Kuester

Abstract: For measure-theoretic dynamical systems a one-dimensional fixed space of the Koopman operator on some L^p-space is equivalent to ergodicity of the underlying system. In the topological case the situation is more complex. Transitivity implies a one-dimensional fixed space in $C(K)$, but the converse is not true in general. In my talk I characterize the fixed space in terms of the dynamics leading to a sufficient and necessary condition for a one-dimensional fixed space.

Spectral enclosures for extensions of symmetric operators

Matthias Langer

Abstract: In this talk I will consider a framework that can be used to describe extensions of symmetric operators in Hilbert spaces. In particular, I will consider elliptic operators with possibly non-self-adjoint and/or non-local boundary conditions and Schrödinger operators with δ-potentials supported on hypersurfaces and with real or complex coefficients. The abstract framework can be used to obtain enclosures for the spectrum or to prove that they generate analytic semigroups.
This talk is based on joint work with Jussi Behrndt, Vladimir Lotoreichik and Jonathan Rohleder.

Long-time behaviour of a kinetic model for annihilation

Bertrand Lods

Abstract: We discuss here the convergence to a self-similar profile of the solution to a Boltzmann equation for kinetic annihilation. Such a model does not exhibit any conservation law. The existence and uniqueness of a self-similar profile ha been obtained in earlier contribution and we prove here the convergence to this profile for long time. The rate of convergence is algebraic. The proof combines a detailed spectral analysis of the linearized operator with entropy production estimates.
This is a joint Work with Ricardo Alonso (PUC-Rio) and Véronique Bagland (UBP, Clermont-Ferrand).

About the asymptotic stability of Markov semigroups

Stéphane Mischler

Abstract: I will discuss some classical and more recent asymptotic stability results about some classes of Markov semigroups. I will in particular present a recent version of the Krein-Rutman theorem and I will revisit the Doeblin-Harris-Meyn-Tweedie-Hairer-Mattingly theory.
Possible extensions to the subgeometric case will be presented.

Analyse spectrale d'opérateurs neutroniques isotropes et partiellement élastiques

Yahya Mohamed

Résumé: Nous faisons la théorie spectrale d'une nouvelle classe d'opérateurs de transport des neutrons dans des domaines bornés avec flux rentrant nul où l'opérateur de collision K tient compte des différents types de collisions (élastiques ou inélastiques) des neutrons avec le milieu ambiant. Nous montrons comment accéder au spectre asymptotique de ces opérateurs neutroniques (la partie du spectre discret qui détermine le comportement asymptotique en temps des problèmes de Cauchy associés). En particulier, nous montrons que cette partie du spectre est d'une part située sur l'axe réel et d'autre part est reliée aux valeurs propres réelles, situées en dehors du disque spectral essentiel, d'opérateurs bornés non compacts et symétrisables par K. Nous montrons aussi le lien de ce spectre asymptotique avec la valeur propre dominante de modèles neutroniques homogènes en espace.

On the linear operator approach to dynamical systems

Rainer Nagel

Abstract: Already J. von Neumann and G. Birkhoff, while proving their famous ergodic theorems around 1930, associated linear operators to a non linear dynamical system. We discuss this global linearization procedure and indicate some recent applications.

L^p- theory of Schrödinger systems

Abdelaziz Rhandi

Abstract: In this talk we study for $p \in (1, \infty)$ the L^p-realization of the vector-valued Schrödinger operator $Lu := \operatorname{div}(Q\nabla u) + Vu$. Using a noncommutative version of the Dore–Venni theorem due to Monniaux and Prüss, we prove that the L^p-realization of L, defined on the intersection of the natural domains of the differential and multiplication operators which form L, generates a strongly continuous contraction semigroup on $L^p\left(\mathbb{R}^d; \mathbb{C}^m\right)$. We also study additional properties of the semigroup such as extension to L^1, positivity and ultracontractivity and prove that the generator has compact resolvent. We end by giving several examples and counterexamples.
This is a joint work with Markus Kunze, Luca Lorenzi and Abdallah Maichine.

Asymptotic decomposition of substochastic semigroups and applications

Ryszard Rudnicki

Abstract: Stochastic semigroups have been intensively studied because they play a special role in applications. They are used to investigate the long-time behaviour of the distributions of Markov processes like diffusion processes and piecewise deterministic processes. We present new results concerning the long-time behaviour of substochastic semigroups [1,2]. We apply these results to study asymptotic stability of semigroups generated by piecewise deterministic Markov processes. We illustrate mathematical results by applications to kinetic equations [3] and gene expression models [4].
[1] K. Pichór, R. Rudnicki, Asymptotic decomposition of substochastic operators and semigroups, J. Math. Anal. Appl. 2016.
[2] K. Pichór, R. Rudnicki, Stochastics and Dynamics 2017.
[3] R. Rudnicki, A. Tomski, On a stochastic gene expression with pre-mRNA, mRNA and protein contribution, J. Theor. Biol. 2015.
[4] R. Rudnicki, M. Mokhtar-Kharroubi, On asymptotic stability and sweeping of collisionless kinetic equations, Acta Appl. Math. 2016.

Cross diffusion equations for non-isothermal gaseous mixtures

Francesco Salvarani

Abstract: The diffusive behavior of multicomponent gaseous mixtures has recently gained interest in the mathematical community. In this talk, I will study a system describing diffusive phenomena for a mixture in a non-isothermal setting. I will prove local existence and uniqueness of the solution and discuss some effects of the temperature on the mixture's behavior.

Existence results for nonlinear transport equations: a fixed point theory approach

Ahmed Zeghal

Abstract: In this talk, we are concerned with several existence results for some nonlinear transport equations arising in the theory of growing cell populations and in neutron transport theory. The solutions are given in L_1 - spaces, which is the convenient and the natural setting of these equations. The problems are first reduced to a fixed point theorem

$$F(\lambda)\psi := (\lambda - T_H)^{-1} K N_f \psi = \psi,$$

where T_H is the nonlinear streaming operator, K is the collision operator and N_f is the Nemytskii operator generated by f. Next, we prove that when the operator K is regular in the sense of M. Mokhtar-Kharoubi [1] and B. Lods [2], then $(\lambda - T_H)^{-1} K$ maps weakly compact sets of L_1 into compact ones in multi-dimensional case (in one dimensional case, it is weakly compact). Finally, we show how this observation together with the fixed point theorem [3, Theorem 2.1] enables us to solve our problems.

References

[1] M. Mokhtar-Kharroubi, *Mathematical Topics in Neutron Transport Theory. New Aspects,* Series on advances in Mathematics for Applied Sciences, Vol. 46 (World Scientific, 1997).

[2] B. Lods, *On linear kinetic equations involving unbounded cross-sections,* Math. Methods Appl. Sci., **27** (2004) 1049-1075.

[3] K. Latrach, M. A. Taoudi and A. Zeghal, *Some fixed point theorems of the Schauder and the Krasnosel'skii type and application to nonlinear transport equations,* J. Differential Equations, **221** (2006) 256-271.