Noncommutative geometry
Arbre de Noël
Besançon, November 27-29, 2014

Abstracts

Philippe Biane (IGM, Universite Paris Est)

Free entropy and noncommutative transport

I will give an introduction to free entropy and explain some of its relations to noncommutative transport.

Francesco DAndrea (Universita di Napoli "Federico II")

Pythagoras Theorem in Noncommutative Geometry

After a brief introduction to the spectral distance and its properties, and its connection with transport theory, I will discuss a version of Pythagoras theorem which holds in noncommutative geometry.

Kenny De Commer (VUB)

Coamenability for partial compact quantum groups

T. Hayashi introduced the notion of 'compact quantum group of face type', which is to be seen as a compact quantum groupoid with a finite object set. In this talk, we introduce the notion of 'partial compact quantum group', which is a generalization of Hayashi's definition to the case of an infinite object set. Partial compact quantum groups can for example be constructed from any rigid tensor C*-category, and from any ergodic action of a compact quantum group. We look at two concrete examples related to the dynamical quantum $SU(2)$ group, and investiage in particular their coamenability. This is joint work with T. Timmermann.

Amaury Freslon (Universität des Saarlandes)

The Bi-Finetti theorem

The De Finetti theorem establishes a link between the invariance of a family of random variables under permutations and their conditional independence. A noncommutative version of this result was proved by C. Kostler and R. Speicher, linking freeness with amalgamation with an action of the quantum permutation group. We will present an analogue of this theorem in the setting of bi-free probability recently introduced by D.V. Voiculescu. The quantum symmetries are again given by quantum permutations, but this time the action has to be twisted using the particuler combinatorics of bi-freeness, inducing algebraic and analytical subtleties.

Piotr Hajac (IMPAN, Warsaw)

Odd-dimensional multi-pullback quantum spheres

We construct a noncommutative deformation of odd-dimensional spheres that preserves the natural partition of the $(2n+1)$-dimensional sphere into $(n+1)$-many solid tori. This generalizes the case $n = 1$ referred to as the Heegaard quantum sphere. Our odd-dimensional quantum sphere C*-algebras are given as multi-pullback C*-algebras. We prove that they are isomorphic to the universal C*-algebras generated by certain isometries, and use this result to compute the K-groups of our odd-dimensional quantum spheres. Furthermore, we prove that the fixed-point subalgebras under the diagonal $U(1)$-action on our quantum sphere C*-algebras yield the independently defined C*-algebras of the quantum complex projective spaces constructed from the Toeplitz cubes. Then, by constructing a strong connection, we show that this $U(1)$-action is free. This leads to the main result stating that the noncommutative line bundles over the quantum complex projective spaces that are associated to this action via non-trivial representations of $U(1)$ are not stably trivial. (Based on joint work with D. Pask, A. Sims and B. Zieliński.)

Frédéric Latrémolière (University of Denver)

The Gromov-Hausdorff Propinquity

Motivated by the quest for an analogue of the Gromov-Hausdorff distance in noncommutative geometry which is well behaved with respect to C*-algebraic structures, we constructed the Gromov-Hausdorff propinquity, a complete metric on the class of Leibnitz quantum compact metric spaces. This metric resolves several important issues raised in the recent research in noncommutative metric geometry: it makes *-isomorphism necessary for distance zero, is well-adapted to Leibniz seminorms, and is complete. Moreover, it admits a generalization to quantum proper metric spaces. We will survey, in this talk, the notion of a quantum metric space, including our notion of a quantum locally compact metric space, and then present our new metric. We will discuss how the propinquity may be adapted to the locally compact quantum metric spaces setting.

Benben Liao (UFC Besancon & UCB Lyon)

Approximation property for almost simple algebraic groups over local fields and their lattices

Let F be a non Archimedean local field of any characteristic (for example $F = \mathbb{Q}_q, \mathbb{F}_q((T))$), $G = Sp_4(F)$ and K a maximal compact subgroup of G. It is shown that $\forall p \in [1, 4/3) \cup (4, \infty]$, compactly supported K-biinvariant completely bounded Schur multipliers on the Schatten p class $S^p(L^2(G))$ decay on G uniformly, provided that their cb-norms are uniformly bounded. As a consequence, together with the results of Lafforgue—de la Salle and Haagerup—de Laat, one concludes that almost simple algebraic groups of split rank ≥ 2 over any local field and their lattices do not have the approximation property of Haagerup and Kraus.

Pierre Martinetti (University of Trieste)

Twisted spectral triple for the standard model

Twisted spectral triples (also called sigma spectral triples) have been introduced by Connes and Moscovici to deal with type 3 algebras. They are also relevant in elementary situations, such as conformal transformation of a Riemannian manifold: commutators with the Dirac operator are unbounded, twisted commutators are bounded. In the standard model of elementary particles, twisting the spectral triple permits to generate as a twisted fluctuation - satisfying a twisted version of the first-order condition - the extra scalar field which is required to both stabilize the electroweak vacuum and make the computation of the Higgs boson mass compatible with its experimental value. Furthermore, the twist opens interesting ways to physics beyond the standard model.

Colin Mrozinski (IMPAN, Warsaw)

Compact semitopological quantum semigroup

We study the notion of compact semitopological quantum semigroup introduced by M. Daws, and we derive sufficiant conditions for a compact semitopological quantum semigroup to be a compact quantum group, generalizing some previous results.

Henrik Petersen (EPF Lausanne)

Quasi-isometries of Nilpotent Groups

Every discrete, finitely generated, torsion free nilpotent group G embeds in a unique connected, simply connected, nilpotent Lie group as a cocompact lattice. This ambient Lie group is called the Mal'cev completion of G. A well-known open question asks whether the Mal'cev completion is invariant under quasi-isometry. In my talk I will present some recent progress in this direction. This is joint work with David Kyed.

Lorenzo Pittau (University of Cergy & Paris 7)

The free wreath product of a discrete group by a quantum automorphism group

The free wreath product of a compact quantum group by a quantum permutation group was defined by J. Bichon in 2004. In 2013 F. Lemeux, generalizing a previous work of T. Banica and R. Vergnioux, studied the free wreath product of a discrete group by a quantum permutation group. We want to give a further generalization of these results considering a quantum automorphism group instead of a quantum permutation group. We start giving a new description of the intertwining spaces of a quantum automorphism group using non-crossing partitions. We can then define the free wreath product of a discrete group by a quantum automorphism group and describe its intertwining spaces using decorated non-crossing partitions. The fusion rules, the irreducible representations and some other properties are so obtained.

Manon Thibault (University of Clermont-Ferrand)

Classification of bicovariant differential calculi over free orthogonal Hopf algebras

After recalling the notion of bicovariant differential calculi over Hopf algebras, and of the equivalent notion formulated in terms of Yetter-Drinfeld modules, I will explain how can be obtained the classification of bicovariant differential calculi over free orthogonal Hopf algebras. An important step in this classification is made by showing that monoidally equivalent Hopf algebras have equivalent categories of bicovariant differential calculi. Since free orthogonal Hopf algebras are monoidally equivalent to $O_q(SL_2)$ for a given q, the problem then reduces to the question of classifying bicovariant differential calculi over $O_q(SL_2)$.

Stefan Vaes (KU Leuven)

Classification of crossed product von Neumann algebras

Actions of countable groups by automorphisms give rise to interesting families of von Neumann algebras through the crossed product construction. Starting from a joint work with Sorin Popa on crossed products by actions of the free groups, I will survey several recent classification theorems for type II_1 factors, type III factors and subfactors, arising from actions on measure spaces, as well as on injective factors.

Simeng Wang (University of Franche-Comté and Polish Academy of Sciences)

L_p-improving convolution operators on finite quantum groups

For A being a finite dimensional C*-algebra equipped with a faithful tracial state τ, and $T : A \to A$ being a unital trace preserving map on A, we prove that the L_p-improving property $\|T : L_p(A) \to L_2(A)\| = 1$ with some $1 < p < 2$ holds if and only if we have the "spectral gap": $\sup_{x\in A\setminus\{0\},\tau(x)=0} \|Tx\|_2/\|x\|_2 < 1$. As a result we characterize positive convolution operators on a finite quantum group $\mathbb{G}$ which are L_p-improving. More precisely, it is proved that the convolution operator $T_\varphi : x \mapsto \varphi \star x$ given by a state φ on $C(\mathbb{G})$ satisfies

$$\exists 1 < p < 2, \quad \|T_\varphi : L_p(\mathbb{G}) \to L_2(\mathbb{G})\| = 1$$

if and only if the Fourier series $\hat{\varphi}$ satisfies $\|\hat{\varphi}(\alpha)\| < 1$ for all nontrivial irreducible unitary representation α, if and only if the state $(\varphi \circ S) \star \varphi$ is non-degenerate (where S is the antipode). We also prove that these L_p-improving properties are stable under taking free products, which gives a method to construct L_p-improving multipliers on infinite compact quantum groups.

Vito Zenobi (Institut Mathematiques de Jussieu, Paris)

Lie Groupoids and secondary invariants

Abstract: The homotopy invariance of the analytic signature and the positive scalar curvature of a Riemannian spin manifold are examples of the analytic index vanishing (for the Signature operator and the Dirac operator respectively). This gives a lifting of the K-homology class, associated to the operator, to a certain K-group. We define these invariants in more complicated geometrical situations (manifolds with corners, foliations, etc...) encoded by a Lie groupoid. These invariants apply to the study of the Structure Set (in the Surgery Exact Sequence) and the space of positive scalar curvature metrics of a Riemannian manifold.