Compact convex sets that admit a strictly convex function

based on an ongoing work with L.C. García-Lirola and J. Orihuela

M. Raja (Murcia)

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Some other technical concepts will be mentioned by specific reasons but not used: fragmentability, Eberlein compact, Namioka-Phelps compact, property $(*)$...

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For instance, cluster points of subsets of *Eberlein compacts* (homeomorphic to a weakly compact subset of a Banach space) can be reached by sequences, among some other remarkable properties.

Convex continuous functions are lower semicontinuous with respect to the weak topology. Moreover, the conditions on a Banach space implying that it has an equivalent strictly convex norm has been investigated during decades.

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Notice that the definition of the class $\mathcal{SC}(X)$ is not purely topological, as it depends on the geometrical structure of K.

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Talagrand (1986)

If $K \in \mathcal{S}C$ then $[0, \omega_1]$ does not embed into K.

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- If $C \in \mathcal{S}C(X)$, then it is witnessed by a bounded strictly convex lower semicontinuous function.
- If $C \in \mathcal{S}C(X)$, then it is witnessed by the square of a lower semicontinuous strictly convex norm defined on $\mathrm{span}(C)$.

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All these results are true for countably many functions simultaneously.

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- K is *uniformly Eberlein* if and only if it embeds into a uniformly convex Banach space endowed with the weak topology.
- K is *Namioka-Phelps* if and only if it embeds into a dual Banach space with a locally uniformly convex norm endowed with the weak* topology.

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Asplund + Larman-Phelps

Let X be a dual Banach space with a dual strictly convex norm. Then every weak* compact subset has an weak* exposed point.

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The existence of continuity extreme points of f implies the following

If $K \subset X$ is compact and convex, and $f : K \to \mathbb{R}$ is strictly convex, bounded and lower semicontinuous then every nonempty subset of K has arbitrarily ρ-small slices.

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The same techniques that in the strictly convex case allows us to obtain faces where the function remains constant.

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We have made use of symmetrics to measure diameters of slices. We can go further this way and consider ordinal indices with respect to the set derivation induced as

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The previous result compares to the following proved in 2008.

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If one of the previous statements holds then C is weakly compact and uniformly Eberlein endowed with the weak topology.

Thank you for your attention!