Extreme differences on the size of weakly open subsets

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Preliminaries

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Another approach

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Copy the Poulsen simplex

Fix $\{\varepsilon_n\} \searrow 0$ and denote's by $\{e_n\}$ the usual basis of c_0.
$K_1 := \{e_1\}$, $g_1 := e_1$, $K_2 := co(e_1, e_1 + e_2)$, choose $l_2 > 1$ and $\{g_2, \dots, g_{l_2}\}$ an ε_2-net in K_2. Also $m_1 := 1$, $m_2 := 2$ and $l_1 := 1$.
If m_n, l_n, K_n and $g_1, \dots g_{l_n}$ have been defined so that $K_n \subset S_{[e_1,\dots,e_{m_n}]}$ and $\{g_{l_{n-1}+1}, \dots, g_{l_n}\}$ is a ε_n-net in K_n, we define

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Thank you!