Septièmes journées "Besançon-Neuchâtel" d'Analyse Fonctionnelle.

Besançon, du 20 au 23 juin 2017.

Main speakers - Abstracts.

Gilles Godefroy (CNRS - Institut de Mathématiques de Jussieu)

"Closed subspaces of L^1 consisting of continuous functions."

In this joint work with N. Lerner, we investigate the Banach space structure of closed subspaces of $L^1(\Omega)$, where Ω is a locally compact space, which consist of continuous functions on Ω. We show for instance that such spaces have the Schur property. The special case of dilation-invariant spaces on a star-shaped domain in $\mathbb{R}^n$, where more information is available, is considered. Numerous examples are provided where such results apply.

Ana Khukro (Université de Neuchâtel)

"Geometry of finite quotients of groups."

Geometric properties of a collection of finite quotients of a group can provide information about the group if the set of finite quotients is sufficiently rich. Using a metric space constructed using Cayley graphs of these finite quotients, one can exploit the connections between the world of group theory and graph theory to give examples of metric spaces with interesting and often surprising properties, as well as learn more about the group itself. In this talk, we will describe some results in this direction, and then give recent results concerning geometric rigidity of finite quotients of a group (joint work with Thiebout Delabie).

Tim de Laat (Universität Münster)

"Obstructions to coarse embeddability of expanders into Banach spaces."

Expanders are sequences of finite sparse graphs with strong connectivity properties. It is well known that expanders do not coarsely embed into Hilbert spaces (or even into uniformly curved spaces). However, the question whether there exists an expander that admits a coarse embedding into a uniformly convex space remains open. In this talk, I will first explain how strengthenings of Kazhdan's property (T) can be used to show that certain Margulis-type expanders, which are expanders that are constructed in a specific way from higher rank lattices, do not embed into various classes of Banach spaces. This is based on fundamental work of Vincent Lafforgue and on joint work of Mikael de la Salle and myself. I will also comment on some very recent work that shows that certain expanders that are not of Margulis-type also have bad embeddability properties.

Masato Mimura (École Polytechnique Fédérale de Lausanne)

"Intrinsic upgrades of fixed points without bounded generation."

"Shalom's Bounded Generation Argument(s)" (1999, Publ. IHES and 2006, ICM) enable us to "upgrade" the existence of fixed points for small subgroups to that for the whole group, just by looking at "intrinsic" structure of the group and the subgroups. We proceed in his direction ; we succeed in removing any form of "Bounded Generation" hypothesis.

Matías Raja Baño (Universidad de Murcia)

"Nonlinear maps motivated by linear properties."

Well known isomorphic properties of Banach spaces, as the Radon-Nikodym property or the super-reflexivity, were already translated into operator language leading to operator ideals. Inspirited by the point continuity property, we propose a version for general maps of the above mentioned properties. The classes of maps so obtained not only enjoy nice properties, further they tie together quite different notions and allow to sharpen previous results. In particular, we characterize the functions that can be uniformly approximated by differences of convex Lipschitz functions, we show relations to $\mathcal{D}C$-maps and we explore the utility of these techniques in optimization.

Christian Rosendal (University of Illinois at Chicago)

"Coarse equivalence, topological couplings and a theorem of Gromov."

A seminal theorem of M. Gromov states that two finitely generated groups are quasi-isometric if and only if they admit a topological coupling, thus establishing a link between the geometry and topological dynamics of groups. Much work has been done recently on expanding the tools and results of geometric group theory to locally compact groups and beyond and we shall explain how Gromov's theorem admits generalisations to all locally compact groups and even a special class of non-locally compact topological transformation groups. (The initial part of the talk will be based on joint work with U. Bader.)

Mikael de la Salle (CNRS - École Normale Supérieure de Lyon)

" Banach space actions and L^2-spectral gap."

Zuk proved that if a finitely generated group admits a Cayley graph such that the Laplacian on the links of this Cayley graph has a spectral gap $> 1/2$, then the group has property (T), or equivalently, every affine isometric action of the group on a Hilbert space has a fixed point. I will explain a recent joint work with Tim de Laat, where we prove that the same holds for affine isometric actions of the group on a uniformly curved Banach space (for example an L^p-space in the reflexivity range or a subquotient of an interpolation space between a Hilbert space and an arbitrary Banach space) as soon as the Laplacian on the links has a two-sided spectral gap $> 1 - \varepsilon$. This applies to Gromov's random groups.

Thomas Schlumprecht (Texas A&M University)

"Not every infinite dimensional Banach space coarsely contains Hilbert space".

In this joint work with Florent Baudier and Gilles Lancien we show that ℓ_2 does not coarsely embed into Tsirelson's original space T^*. We will deduce that result from a concentration inequality for Lipschitz maps on the infinite Hamming graph with values in T^*.

Romain Tessera (CNRS - Université Paris-Sud)

"Gromov's monsters do not act on hyperbolic spaces."

We show that the monster groups arising from Gromov's random construction do not admit non-elementary actions on hyperbolic spaces. The proof relies on comparing properties of random walks on randomly labeled graphs and on groups acting non-elementarily on hyperbolic spaces. This is a joint work with Dominik Gruber and Alessandro Sisto.

Short talks - Abstracts.

Catalin Badea (Université de Lille)

"Groups with property (T) : an operator-theoretic point of view."

Continuing our previous work with Sophie Grivaux, we present some results about groups having property (T) from an operator theoretical perspective.

Christian Bargetz (Universität Innsbruck)

"A Fréchet space of almost universal disposition."

The Gurariĭ space is the unique separable Banach space $\mathbb{G}$ which is of almost universal disposition for finite-dimensional Banach spaces, which means that for every $\varepsilon > 0$, for all finite-dimensional normed spaces $E \subseteq F$, for every isometric embedding $e\colon E \to \mathbb{G}$ there exists an ε-isometric embedding $f\colon F \to \mathbb{G}$ such that $f \restriction E = e$.
S. Mazur and W. Orlicz showed that the space $\mathcal{C}(\mathbb{R})$ of continuous functions on the real line is universal in the class of separable Fréchet spaces and therefore is in some sense the equivalent of $\mathcal{C}([0,1])$ for Fréchet spaces since $\mathcal{C}([0,1])$ has the same property in the class of separable Banach spaces.
It seems natural to ask whether there is an equivalent of the Gurariĭ space for separable Fréchet spaces, i.e. whether there is a separable Fréchet space which is of almost universal disposition for finite-dimensional Fréchet spaces. We show that there is a sequence of semi norms on the countable product $\mathbb{G}^{\mathbb{N}}$ with which it is of almost universal disposition for finite-dimensional graded Fréchet spaces. The construction relies heavily on the universal operator on the Gurariĭ space, recently constructed by J. Garbulińska-Węgrzyn and W. Kubiś.
This is joint work with Jerzy Kąkol and Wiesław Kubiś.

Marek Cúth (Charles University - Prague)

"Finitely additive measures and complementability of Lipschitz-free spaces."

Given a metric space M it is possible to construct a Banach space $F(M)$ in such a way that the metric structure of M corresponds to the linear structure of $F(M)$. This space $F(M)$ is sometimes called the Lipschitz-free space over M. The study of Lipschitz-free spaces is well-motivated : for example, if we knew that $F(\ell_1)$ is complemented in its bidual, it would solve famous open problem of whether every Banach space which is Lipschitz-isomorphic to ℓ_1 is actually linearly isomorphic to ℓ_1. I will talk about our recent paper with O. Kalenda and P. Kaplický, where we prove that $F(\mathbb{R}^n)$ is complemented in its bidual.

Willian Corrêa (Universidade de São Paulo)

"Twisting Operator Spaces."

Complex interpolation induces an extension, or twisted sum, of the interpolated space. We show how this construction may be naturally carried to the operator space scenario, and show the existence of a hilbertian operator space not completely isomorphic to the operator Hilbert space OH which has a subspace completely isomorphic to OH with respective quotient also completely isomorphic to OH.

Wilson Cuellar Carrera (Universidade de São Paulo)

"About ergodic and complementably universal Banach spaces."

The notion of ergodic Banach spaces was introduced by V. Ferenczi and C. Rosendal in 2005 to study the problem of the complexity of the relation of isomorphism between the subspaces of a separable Banach space not isomorphic to ℓ_2 by using methods of descriptive set theory. We obtain a criterion for ergodicity for separable Banach spaces. In particular, we prove that a Banach space that is not ergodic is necessarily near Hilbert, which reinforces the Ferenczi-Rosendal conjecture that ℓ_2 is the only Banach space that is not ergodic. The methods we use also relate to universal complementably Banach spaces.

Loïc Demeulenaere (Université de Liège)

"Diametral dimension(s) and prominent bounded sets."

The classical diametral dimension (Bessaga, Mityagin, Pełczyński, Rolewicz), denoted by Δ, is a topological invariant which can be used to characterize Schwartz and nuclear locally convex spaces. Mityagin also introduced a variant of this diametral dimension, denoted by Δ_b, using bounded sets in its definition, contrary to Δ. In this talk, we present some conditions assuring the equality of these two diametral dimensions for Fréchet spaces. Among these conditions, there is the notion of existence of prominent bounded sets, due to Terzioğlu. In fact, it appears that the existence of prominent sets is implied by the property $\left(\overline{\Omega}\right)$ of Vogt and Wagner. Finally, we describe a construction which gives Schwartz and nuclear non-Fréchet spaces E verifying $\Delta_b(E) \neq \Delta(E)$.

Michal Doucha (Czech Academy of Sciences)

"Global aspects of unitary representations."

For a fixed countable group G we investigate the space of all unitary representations of G in an infinite-dimensional separable Hilbert space. We show that whenever G has the Haagerup property and its full group C* algebra is residually finite-dimensional, then the conjugacy class of any unitary representation of G is meager. That extends the known results for free groups. On the other hand, when G has the Kazhdan property and its group C* algebra is residually finite-dimensional, then some yet unproved, nevertheless a plausible condition on positive definite functions associated to irreducible finite-dimensional representations of G gives that G has a unitary representation with a comeager conjugacy class. Using a certain Baire category argument we can then conclude that G is finite. Thus the unproved condition would lead to a negative answer to the question of Lubotzky and Shalom whether there is a Kazhdan group with property FD. This is joint work with Maciej Malicki.

Michael Dymond (Universität Innsbruck)

"Mapping n grid points onto a square forces an arbitrarily large Lipschitz constant."

We prove that the regular $n \times n$ square grid of points in the integer lattice $\mathbb{Z}^2$ cannot be recovered from an arbitrary n^2-element subset of $\mathbb{Z}^2$ via a mapping with prescribed Lipschitz constant (independent of n). This answers negatively a question of Feige from 2002. Our resolution of Feige's question takes place largely in a continuous setting and is based on new results for Lipschitz mappings falling into two broad areas of interest, which we study independently. Firstly we discuss *Lipschitz regular mappings* on Euclidean spaces, with emphasis on their bilipschitz decomposability in a sense comparable to that of the well known result of Jones. Secondly,

we build on work of Burago and Kleiner and McMullen on *non-realisable densities*. We verify the existence, and further prevalence, of strongly non-realisable densities inside spaces of continuous functions. This is joint work with Vojtěch Kaluza and Eva Kopecká.

Luis Carlos García Lirola (Universidad de Murcia)

"A characterisation of the Daugavet property in spaces of Lipschitz functions."

The aim of this talk is to characterise which spaces of Lipschitz functions $\mathrm{Lip}_0(M)$ enjoy the Daugavet property in terms of a geometric condition on the underlying metric space M. Namely, we show that $\mathrm{Lip}_0(M)$ has the Daugavet property if, and only if, M is a length space. This condition also characterises when the Lipschitz-free space $\mathcal{F}(M)$ has the Daugavet property. Moreover, in the case of a compact metric space, we show that either $\mathcal{F}(M)$ has the Daugavet property or its unit ball has a strongly exposed point. If M is a compact subset of a rotund Banach space then the Daugavet property of $\mathrm{Lip}_0(M)$ is equivalent to the convexity of M. This is part of a joint work with A. Procházka and A. Rueda-Zoca.

Jason Hancox (Lancaster University)

"Isometry C*-bialgebras and their Lévy processes."

We will introduce C*-bialgebras and Lévy Processes on C*-bialgebras. These are a natural generalization of compact topological semigroups and stochastic processes on compact topological semigroups that have "independent increments" respectively. We will construct a family of C*-bialgebras which we call the isometry C*-bialgebras and characterize the Lévy Processes on this family.
To finish we will consider the Toeplitz algebra and investigate some processes on its C*-bialgebra structure and find concrete realizations of these processes on a commutative quotient subalgebra.

Jan-David Hardtke (Freie Universität Berlin)

"On certain geometric properties in Banach spaces of vector-valued functions."

We will consider a certain type of geometric properties of Banach spaces, which are defined in an abstract way via families of so called "test functions". These properties include for instance octahedrality, almost squareness, lushness and the Daugavet property.
We will discuss a general theorem stating that if such a property is stable with respect to certain finite absolute sums (for eaxmple finite ℓ^p-sums), then it is also stable with respect to the formation of corresponding Köthe-Bochner spaces of vector-valued functions (for example L^p-Bochner spaces).
This abstract result will then be applied to show some statements on specific geometric properties in Köthe-Bochner spaces.

James Kilbane (University of Cambridge)

"On embeddings of finite subsets of ℓ_p."

One of the most important theorems in the local theory of Banach spaces is Krivine's theorem : for any infinite-dimensional Banach space there is some p such that the space contains almost isometric copies of l_p^n. In this talk we shall consider a metric version of this result, and investigate whether finite subsets of l_p can be isometrically embedded into our space.

Ondřej Kurka (Charles University and Czech Academy of Sciences)

"Tsirelson-like spaces and complexity questions in Banach space theory."

We will present an approach to complexity problems in Banach space theory which is based on the fundamental example of Tsirelson.

Sebastián Lajara (Universidad de Castilla-La Mancha)

"Operator ranges and quasi-complemented subspaces of Banach spaces."

Given bounded linear operators $A : X \to E$ and $T : E \to Y$ between Banach spaces, with E separable, and a subspace $L \subset E$ such that $L \cap AX = \{0\}$, we provide sufficient conditions to ensure the existence of an infinite codimensional subspace $L_1 \subset E$ such that $L \subset L_1$, $L_1 \cap AX = \{0\}$ and $\operatorname{cl} TL_1 = \operatorname{cl} TE$. Some applications to the study of quasi-complemented subspaces of a Banach space are also given. This talk is based on a joint work with V. P. Fonf, S. Troyanski and C. Zanco.

Soyoung Moon (Université Bourgogne Franche-Comté)

"Homogeneous actions on Random Graph."

Let $\mathcal{R}$ be the Random Graph (or Rado graph). A group action $\Gamma \curvearrowright \mathcal{R}$ is *homogeneous* if, for any graph isomorphism $\varphi : U \to V$ between finite induced subgraphs U, V of $\mathcal{R}$, there exists $g \in \Gamma$ such that $g(u) = \varphi(u)$ for all $u \in U$. In this talk, we will discuss the class of countable groups that admit a faithful and homogeneous action on $\mathcal{R}$. I will explain, under some conditions, how to construct such an action for groups acting on trees. This is joint work with Pierre Fima and Yves Stalder.

Matěj Novotný (Czech Technical University, Czech Academy of Sciences)

"Distortion of Lipschitz functions on $c_0(\Gamma)$."

Let Γ be an uncountable cardinal. We construct a real symmetric 1-Lipschitz function on the unit sphere of $c_0(\Gamma)$ whose restriction to any nonseparable subspace is distorted. On the other hand, every equivalent renorming of $c_0(\Gamma), \ell_1(\Gamma)$ is oscillation stable (in the nonseparable sense).

Eva Pernecká (Institute of Mathematics, Polish Academy of Sciences)

"Weak sequential completeness of Lipschitz-free spaces."

The talk is based on a joint work with Tomasz Kochanek. Extending an earlier result due to Cúth, Doucha and Wojtaszczyk, we show that for every compact subset M of a superreflexive Banach space, the Lipschitz-free space $\mathcal{F}(M)$ is weakly sequentially complete. So, in particular, it does not contain a copy of c_0. Our approach is based on adapting the proof of Bourgain's result about the weak sequential completeness of $(C^1([0,1]^n))^*$ and combining it with combinatorial properties of superreflexive spaces as well as certain approximation techniques for Lipschitz maps.

Colin Petitjean (Université Bourgogne Franche-Comté)

"Lipschitz-free spaces and Schur properties."

Consider a metric space M with a distinguished point 0_M. Let $Lip_0(M)$ be the set of Lipschitz functions from M to $\mathbb{R}$ satisfying $f(0_M) = 0$. The Lipschitz-free space over M is defined as follows :

$$\mathcal{F}(M) := \overline{\text{span}}^{\|\cdot\|}\{\delta_M(x) \,:\, x \in M\} \subset Lip_0(M)^*,$$

where $\delta_M(x)$ is the evaluation map at x. The fundamental factorization property of Lipschitz-free spaces transforms a nonlinear problem into a linear one. Indeed, if X is a real Banach space, then the space $Lip_0(M, X)$ (of X-valued Lipschitz functions vanishing at 0_M) is identified in a canonical way with $\mathcal{L}(\mathcal{F}(M), X)$. This creates links between old open problems in the geometry of Banach spaces and open questions about Lipschitz-free spaces. After the seminal paper of Godefroy and Kalton [1], Lispchitz-free spaces became an object of interest for many authors. The aim of most of the papers is to explore the linear structure of Lipschitz-free spaces looking for properties such as approximation properties, weak sequential completeness, etc. In this presentation, we focus on the study of the Schur property and some strenghtening versions of this property. In particular, we provide conditions on M to ensure that $\mathcal{F}(M)$ has the Schur property, the 1-strong Schur property, or even better in some cases. Next, we introduce a natural version of Lipschitz-free spaces in the vector valued case. More precisely, this vector-valued version $\mathcal{F}(M, X)$ is defined as the canonical predual of $\mathcal{L}(\mathcal{F}(M), X^*)$, namely the projective tensor product of $\mathcal{F}(M)$ and X. We finish this presentation studying again the Schur properties on $\mathcal{F}(M, X)$.

References.

[1] G. Godefroy and N.J Kalton, *Lispchitz-free Banach spaces*. Studia Math. 159, 2003.

[2] C. Petitjean, *Lipschitz-free spaces and Schur properties*, to appear in J. of Math. Anal. Appl.

[3] L. García-Lirola, C. Petitjean and A. Rueda Zoca, *On the structure of spaces of vector-valued Lipschitz functions*, to appear in Studia Math.

Abraham Rueda Zoca (Universidad de Granada)

"Preservance of octahedrality and strong diameter two property by tensor product spaces."

A Banach space X is said to have the *slice diameter two property* (respectively *diameter two property, strong diameter two property*) whenever every slice (respectively non-empty relatively weakly open subset, convex combination of slices) of the unit ball has diameter two. In 2013, T. Abrahamsen, V. Lima and O. Nygaard asked how the above properties are preserved by taking tensor products. In 2015, it was proved that a projective tensor product space has the strong diameter two property whenever both factors enjoy this property, but it remained the open problem of whether the strong diameter two property is stable by projective tensor product from one factor. In this talk I will present a negative answer to this question, which provides a complete answer to the question of how the strong diameter two property is preserved by taking projective tensor product. For this, we will analyse this question in a predual setting, that is, we will establish some necessary and sufficient conditions on an injective tensor product of the form $L_1([0,1])\widehat{\otimes}_\varepsilon Y$ to have an octahedral norm. The presented results are part of a recent work joint to J. Langements and V. Lima.

Reference.

[1] J. Langemets, V. Lima and A. Rueda Zoca, *Octahedral norms in tensor product of Banach spaces*, to appear in the Quarterly J. Math.

Tommaso Russo (Università degli Studi di Milano)

"Some remarks on smooth renormings of Banach spaces."

We prove that in every separable Banach space X with a Schauder basis and a C^k-smooth norm it is possible to approximate, uniformly on bounded sets, every equivalent norm with a C^k-smooth one in a way that the approximation is improving as fast as we wish on the elements depending only on the tail of the Schauder basis. This result is a joint work with Prof. Petr Hájek. Our result solves a problem from the recent monograph of Guirao, Montesinos and Zizler.

Jarno Talponen (University of Eastern Finland)

"Decomposing varying exponent L^p norms in the sense of Nakano by ODE techniques."

It is shown how the Nakano L^p-spaces can be represented as a varying ℓ_p style direct sum decomposition in a natural way, independent of the number of summands. The main tool is an ODE-determined norm which was recently introduced by the author. The key ingredients of the decomposition are some inherent properties of the ODE-determined norm which can be seen by studying the differential equation in question, as well as the equivalence of the ODE-determined norm with the Nakano norm.