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> Conference
on geometric
functional analysis
and its applications

Main speakers

- > **Fernando Albiac**
Universidad P. de Navarra
- > **Florent Baudier**
University of Texas A&M
- > **Alejandro Chavez-Dominguez**
University of Texas at Austin
- > **Robert Deville**
Université de Bordeaux
- > **Stephen Dilworth**
University of South Carolina
- > **Valentin Ferenczi**
Universidade de Sao Paulo
- > **Bill Johnson**
University of Texas A&M
- > **Denka Kutzarova**
*Bulgarian Academy of Sciences
and University of Illinois*
- > **Pavlos Motakis**
University of Athens
- > **Beata Randrianantoanina**
Miami University Ohio
- > **Yves Raynaud**
Université Paris 6 - CNRS
- > **Gideon Schechtman**
Weizmann Institute
- > **Thomas Schlunprecht**
University of Texas A&M
- > **Jarno Talponen**
University of Eastern Finland
- > **Alain Valette**
Université de Neuchâtel



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Conseil régional

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1 Daily schedule

Monday October 27

9:00 - 10:00 : Registration and reception.

10:00 - 10:15 : Conference opening.

10:15 - 11:15 : **Bill Johnson**

Some unrelated results in non separable Banach spaces

11:30 - 12:30 : **Fernando Albiac**

On the fundamental theorem of calculus in the lack of local convexity

12:30 - 14:00 : Lunch break.

14:00 - 15:00 : **Alejandro Chávez-Domínguez**

Lipschitz p -convex and q -concave maps

15:15 - 15:45 : **Christina Brech**

On uncountable biorthogonal structures

15:45 - 16:15 : Coffee break.

16:15 - 16:45 : **Aude Dalet**

Free spaces over proper ultrametric spaces

17:00 - 17:30 : **Eva Pernecká**

Approximation properties in Lipschitz-free spaces

Tuesday October 28

9:15 - 10:15 : **Pavlos Motakis**

The stabilized set of p 's in Krivine's theorem can be disconnected

10:15 - 10:45 : Cofee break.

10:45 - 11:45 : **Gideon Schechtman**

Metric X_p inequalities

12:00 - 12:30 : **Sheng Zhang**

Coarse Quotient Mappings between Metric Spaces

12:30 - 14:00 : Lunch break.

14:00 - 15:00 : **Thomas Schlumprecht**

The algebra of bounded linear operators on $\ell_p \oplus \ell_q$
has infinitely many closed ideals

15:15 - 15:45 : **Matias Raja**

Compact convex sets admitting a strictly convex function

15:45 - 16:15 : Coffee break.

16:15 - 17:15 : **Yves Raynaud**

Banach ultraroots of certain Banach lattices

Wednesday October 29

9:15 - 10:15 : **Florent Baudier**

Metric geometry of stable metric spaces and applications

10:15 - 10:45 : Cofee break.

10:45 - 11:45 : **Valentin Ferenczi**

On singular twistings of Banach spaces

12:00 - 12:30 : **Wilson Cuellar Carrera**

Compatible complex structures on Kalton-Peck space.

12:30 - 14:00 : Lunch break.

15:00 - 17:00 : Guided tour through Besançon. Meeting point in front of the hotel Adagio.

Thursday October 30

9:15 - 10:15 : **Jarno Talponen**

Smoothness of quasihyperbolic balls in convex domains of Banach spaces

10:15 - 10:45 : Cofee break.

10:45 - 11:45 : **Denka Kutzarova**

Lenses and Asymptotic Midpoint Uniform Convexity

12:00 - 12:30 : **Ginés López-Pérez**

Extreme differences on the size of weakly open subsets

12:30 - 14:00 : Lunch break.

14:00 - 15:00 : **Stephen Dilworth**

Greedy bases and the greedy constant

15:15 - 15:45 : **Dong Hoon Cho**

Bishop-Phelps-Bollobas theorem on bounded convex closed sets

15:45 - 16:15 : Coffee break.

16:15 - 16:45 : **Penumarthy Parvateesam Murthy**

n -tupled coincidence point theorems in Probabilistic Menger Spaces

17:00 - 17:30 : **Deepak Singh**

Some Best Proximity Point Theorems in G - Metric Spaces

20:00 - . . . : Conference Dinner. “Bistrot La Charrette” (downtown, 11 rue Jean Petit, tram stop “Révolution”)

Friday October 31

9:15 - 10:15 : **Beata Randrianantoanina**

On an isomorphic Banach-Mazur rotation problem
and maximal norms in Banach spaces

10:15 - 10:45 : Cofee break.

10:45 - 11:45 : **Alain Valette**

The Kadison-Singer Problem

12:00 - 12:30 : **Masato Mimura**

Group approximation in Cayley topology and coarse geometry:
fibered coarse embedding

12:30 - 14:00 : Lunch break.

14:00 - 14:30 : **Tomasz Kobos**

Minimal projections onto hyperplanes in finite dimensional normed space

14:45 - 15:15 : **David Salas**

The Faces-Radon-Nikodým property in Banach spaces

15:15 - 15:45 : Coffee break.

15:45 - 16:45 : **Robert Deville**

Lipschitz embedding of metric spaces into c_0

2 Plenary talks

ON THE FUNDAMENTAL THEOREM OF CALCULUS IN THE LACK OF LOCAL CONVEXITY

Fernando Albiac

Motivated by the attempt to develop tools that can be applied to the study of the nonlinear geometry of quasi-Banach spaces, we embark on a journey to explore which integration methods work for functions mapping in quasi-Banach spaces and investigate the interplay of integration and differentiation in this setting.

METRIC GEOMETRY OF STABLE METRIC SPACES AND APPLICATIONS

Florent Baudier

The Lipschitz, coarse, uniform, and strong geometries of stable metric spaces shall be browsed. Recent developments on embeddability of proper metric spaces will be discussed, in particular a strengthened notion of strong embeddability will be introduced. We will also present applications, or connections, of the above with geometric group theory and topology. Part of this work was carried out in collaboration with G. Lancien.

LIPSCHITZ p -CONVEX AND q -CONCAVE MAPS

Alejandro Chávez-Domínguez

The notions of p -convexity and q -concavity are mostly known because of their importance as a tool in the study of isomorphic properties of Banach lattices, but they also play a role in several results involving linear maps between Banach spaces and Banach lattices. In this talk we introduce Lipschitz versions of these concepts, dealing with maps between metric spaces and Banach lattices, and start by proving nonlinear versions of two well-known factorization theorems through L_p spaces due to Maurey/Nikishin and Krivine. We also show that a Lipschitz map from a metric space into a Banach lattice is Lipschitz p -convex if and only if its linearization is p -convex. Furthermore, we elucidate why there is such a close relationship between the linear and nonlinear concepts by proving characterizations of Lipschitz p -convex and Lipschitz q -concave maps in terms of factorizations through p -convex and q -concave Banach lattices, respectively, in the spirit of the work of Raynaud and Tradacete.

Robert Deville

Let (M, d) be a separable metric space and $1 < \lambda \leq 2$. If E is a non empty subset of $M \times M$, we denote \tilde{E} the smallest rectangle containing E , $\delta(E) := \inf\{d(x, y); (x, y) \in E\}$ and $\text{diam}(E) = \sup\{d(x, y); (x, y) \in E\}$. Following G. Lancien and N. Kalton, we say that (M, d) has the property $\Pi(\lambda)$ if, for all balls B_1 and B_2 of M with radii r_1 and r_2 and for every non empty subset E of $B_1 \times B_2$ such that $\delta(E) > \lambda(r_1 + r_2)$, we can partition E into finitely many subsets E_1, \dots, E_N satisfying

$$\text{for each } n, \quad \text{diam}(E_n) < \lambda\delta(\tilde{E}_n)$$

We prove that property $\Pi(\lambda)$ is equivalent to the existence of a mapping $f = (f_n) : M \rightarrow c_0$ such that for all $x \neq y \in M$, we have $d(x, y) < \|f(x) - f(y)\|$, and there exists a sequence (λ_n) of scalars satisfying, for all n , $\lambda_n < \lambda$ and f_n is λ_n -Lipschitzian.

GREEDY BASES AND THE GREEDY CONSTANT

Stephen Dilworth

A greedy basis is one for which the greedy algorithm yields the best n -term approximation up to a multiplicative constant called the greedy constant. Greedy bases are characterized as being unconditional and ‘democratic’, where the latter is a symmetry condition on constant coefficient vectors. We review the theory of greedy bases, including duality and ‘bidemocratic bases’ and recent results on improving the greedy and democratic constants by renorming.

ON SINGULAR TWISTINGS OF BANACH SPACES

Valentin Ferenczi

We obtain sufficient conditions for singularity of a twisted sum $X \oplus_{\Omega_\theta} X$ induced by an interpolation scheme $X = (X_0, X_1)_\theta$. As consequences we recover and generalize the singularity of Kalton-Peck sums of sequence spaces; discover the ‘disjoint’ singularity of Kalton-Peck sums of function spaces; and also construct singular sums without the use of a lattice structure. Joint work with J. Castillo and M. González.

Bill Johnson

I'll discuss three recent papers, one with Gideon Schechtman, the second with Amir Bahman Nasser, Schechtman, and Tomasz Tkocz, the third with Tomasz Kania and Schechtman. The papers are unrelated but connected in the sense that they are all concerned with the structure of classical non separable Banach spaces; namely, L_p , ℓ_∞ , and ℓ_∞^c (the space of bounded functions that have countable support), respectively.

Denka Kutzarova

As a continuation of a paper on property (β) of Rolewicz we consider a geometric property of the unit ball of a Banach space, namely when the Kuratowski index of noncompactness of some special type of lenses tends to zero uniformly in the elements of the unit sphere. We give an analytic characterization of this new property (AMUC) and show that isometrically it is weaker than asymptotic uniform convexity (AUC). Let us point out that if the diameters of these lenses tend uniformly to zero, this characterizes uniform convexity (Laakso, Tyson-Wu). Finally, we show that an AMUC space with an unconditional basis admits an equivalent AUC norm. This is a joint work with S.J. Dilworth, N. Lovasoa Randrianarivony, J.P. Revalski and N.V. Zhivkov.

Pavlos Motakis

J. L. Krivine's theorem states that for every Banach space X with a basis, there exists a $p \in [1, \infty]$ such that ℓ_p is finitely block represented in X . The set of all such p 's is called the Krivine set of X . As it was proved by H.P. Rosenthal, this set is stabilized on some block subspace Y of X , i.e. the Krivine set of Y and the corresponding one of any of its further block subspaces coincide. The form of such a stabilized Krivine set has been a subject of study, since Rosenthal asked whether it always had to be a singleton. This question was answered negatively by E. Odell and Th. Schlumprecht by constructing a space having $[1, \infty]$ as its stabilized Krivine set. The question that followed was if such a stabilized Krivine set had to be an interval, which was asked by P. Habala and N. Tomczak-Jaegermann as well as by E. Odell. We answer this question in the negative direction by constructing, for every $F \subset [1, \infty]$ which is either finite or consists of an increasing sequence and its limit, a reflexive Banach space X with an unconditional basis such that for every infinite dimensional block subspace Y of X , the Krivine set of Y is precisely F . This construction also addresses some open problems concerning spreading models.

This is a joint work with K. Beanland and D. Freeman.

ON AN ISOMORPHIC BANACH-MAZUR ROTATION PROBLEM AND MAXIMAL
NORMS IN BANACH SPACES

Beata Randrianantoanina

We prove that the spaces ℓ_p , $1 < p < \infty, p \neq 2$, and all infinite-dimensional subspaces of their quotient spaces do not admit equivalent almost transitive renormings. This is a step towards the solution of the Banach-Mazur rotation problem, which asks whether a separable Banach space with a transitive norm has to be isometric or isomorphic to a Hilbert space. We obtain this as a consequence of a new property of almost transitive spaces with a Schauder basis, namely we prove that in such spaces the unit vector basis of ℓ_2^2 belongs to the two-dimensional asymptotic structure and we obtain some information about the asymptotic structure in higher dimensions.

Further, we prove that the spaces ℓ_p , $1 < p < \infty, p \neq 2$, have continuum different renormings with 1-unconditional bases each with a different maximal isometry group, and that every symmetric space other than ℓ_2 has at least a countable number of such renormings. On the other hand we show that the spaces ℓ_p , $1 < p < \infty, p \neq 2$, have continuum different renormings each with an isometry group which is not contained in any maximal bounded subgroup of the group of isomorphisms of ℓ_p .

Joint work with S.J. Dilworth.

BANACH ULTRAROOTS OF CERTAIN BANACH LATTICES

Yves Raynaud

Let \mathcal{C} be an axiomatizable class of Banach lattices, that is, this class is closed under Banach lattice isomorphisms and ultraproducts, and the complementary class is closed under ultrapowers. We show that if linear isometric embeddings of members of \mathcal{C} in their ultrapowers preserve disjointness, the class $\mathcal{C}^{\mathcal{B}}$ of Banach spaces obtained by forgetting the Banach lattice structure is still axiomatizable. Moreover if \mathcal{C} coincides with its “script class” \mathcal{SC} , so does $\mathcal{C}^{\mathcal{B}}$ with $\mathcal{SC}^{\mathcal{B}}$. This allows to give new examples of axiomatizable classes of Banach spaces.

METRIC X_p INEQUALITIES

Gideon Schechtman

A new nonlinear inequality of the flavour of the nonlinear version of the inequalities for type and cotype will be presented. This is a nonlinear extension of a linear inequality that was proved by Johnson, Maurey, Schechtman and Tzafriri in 1979 (and resembles the X_p inequality of Rosenthal). The formulation (and proof) of the new inequality completes the search for bi-Lipschitz invariants that serve as an obstruction to the embeddability of L_p spaces into each other, the previously understood cases of which were metric notions of type and cotype, which however fail to certify the nonembeddability of L_q into L_p when $2 < q < p$. Among the consequences of the new inequality are new quantitative restrictions on the bi-Lipschitz embeddability into L_p of snowflakes of L_q and integer grids in ℓ_q^n , for $2 < q < p$.

Joint work with Assaf Naor.

THE ALGEBRA OF BOUNDED LINEAR OPERATORS ON $\ell_p \oplus \ell_q$ HAS INFINITELY MANY CLOSED IDEALS

Thomas Schlumprecht

In this joint work with A.Zsak, we solve a problem stated by A. Pietsch and show that the algebra of bounded linear operators on $\ell_p \oplus \ell_q$ contains infinitely many closed ideals. More precisely, we show that there is a chain of sub ideals whose cardinality is that of the continuum.

SMOOTHNESS OF QUASIHYPHERBOLIC BALLS IN CONVEX DOMAINS OF BANACH SPACES

Jarno Talponen

Quasihyperbolic (QH) metric is a weighted metric on a path-connected metric space motivated by looking at invariants of Möbius transformations of the unit disk. It is used for analyzing quasiconformal mappings.

In this talk the metric is given on an open convex non-trivial subset of a Banach space. Many properties of the underlying Banach space are visible in the QH geometry.

The study of the following problem was initiated by F.W. Gehring and M. Vuorinen in the 1970s. Is a quasihyperbolic ball $B_{qh}(x, r)$ in a convex domain $\Omega \subset \mathbb{R}^n$ necessarily C^1 -smooth? It turns out that the answer is affirmative for uniformly smooth Banach spaces in place of \mathbb{R}^n . To our knowledge the question was open even for \mathbb{R}^2 and in fact some people conjectured that the answer should be negative for a strip of a plane. There is a preprint in the ArXiv.

THE KADISON–SINGER PROBLEM

Alain Valette

In 1959, R.V. Kadison and I.M. Singer asked whether each pure state of the algebra of bounded diagonal operators on ℓ^2 admits a unique state extension to $B(\ell^2)$. The positive answer was given in June 2013 by A. Marcus, D. Spielman and N. Srivastava, who took advantage of a series of translations of the original question, due to C. Akemann, J. Anderson, P. Casazza, N. Weaver, . . . Ultimately, the problem boils down to an estimate of the largest zero of the expected characteristic polynomial of the sum of independent random variables taking values in rank-one positive matrices in the algebra of n -by- n matrices. In turn, this is proved by studying a special class of polynomials in d variables, the so-called real stable polynomials. The talk will highlight the main steps in the proof.

3 Short talks

ON UNCOUNTABLE BIORTHOGONAL STRUCTURES

Christina Brech

In this talk we will show that the axiom known as P-ideal dichotomy implies that the cardinal inequality $\mathfrak{b} > \omega_1$ is equivalent to the existence of uncountable almost biorthogonal systems in every nonseparable Banach space. This is a joint work with S. Todorćević.

BISHOP-PHELPS-BOLLOBAS THEOREM ON BOUNDED CONVEX CLOSED SETS

Dong Hoon Cho

Since the Bishop-Phelps-Bollobás theorem for operators between Banach spaces has been introduced in 2008, a lot of pairs of Banach spaces were found to satisfy the Bishop-Phelps-Bollobás property. In this talk, we deal with the *Bishop-Phelps-Bollobás property* (*BPBP* for short) on bounded closed convex subsets of a Banach space X , not just on its closed unit ball B_X . We firstly prove that the *BPBP* holds for bounded linear functionals on arbitrary bounded closed convex subsets of a real Banach space. We show that for all finite dimensional Banach spaces X and Y the pair (X, Y) has the *BPBP* on every bounded closed convex subset D of X , and also that for a Banach space Y with property (β) the pair (X, Y) has the *BPBP* on every bounded closed absolutely convex subset D of an arbitrary Banach space X . For a bounded closed absorbing convex subset D of X with positive modulus convexity we get that the pair (X, Y) has the *BPBP* on D for every Banach space Y . We further obtain that for an Asplund space X and for a locally compact Hausdorff L , the pair $(X, C_0(L))$ has the *BPBP* on every bounded closed absolutely convex subset D of X . Finally we study the stability of the *BPBP* on a bounded closed convex set for the ℓ_1 -sum or ℓ_∞ -sum of a family of Banach spaces.

Wilson Cuellar Carrera

The Kalton-Peck space Z_2 is a twisted sum of ℓ_2 with ℓ_2 associated to a singular quasi-linear map Ω_2 . It exhibits very interesting properties. For example, Z_2 has an unconditional FDD consisting of 2-dimensional spaces; nevertheless it has no unconditional basis and no local unconditional structure. An old conjecture (still open) about the Kalton-Peck space is that it's not isomorphic to its hyperplanes. In order to distinguish between Z_2 and its hyperplanes, we wonder if hyperplanes admit complex structures. Recall that a real Banach space X is said to admit a complex structure if there exists an \mathbb{R} -linear operator I on X such that $I^2 = -id$. When such operators exist, we can give a \mathbb{C} -linear structure on X by setting a complex scalar multiplication as follows:

$$(\alpha + i\beta)x = \alpha x + \beta I(x) \quad \forall \alpha, \beta \in \mathbb{R}, \quad \forall x \in X.$$

Endowed with the equivalent norm $\|x\| = \sup_{0 \leq \theta \leq 2\pi} \|e^{i\theta} x\|$ we obtain a complex Banach space.

We say that a complex structure u on ℓ_2 is compatible with Z_2 if there exists a complex structure U on Z_2 such that the restriction of U to the canonical copy of ℓ_2 in Z_2 is u . In this work we prove that there exists a complex structure on ℓ_2 which is not compatible with Z_2 . We also study a notion of compatible complex structure on hyperplanes of Z_2 and prove that hyperplanes of Z_2 do not admit compatible complex structures.

This is a joint work with professors J. M. F. Castillo, V. Ferenczi and Y. Moreno.

References

- [1] N.J. Kalton and N.T. Peck, Twisted sums of sequence spaces and the three space problem, *Trans. Amer. Math. Soc.* 255 (1979) 1–30.
- [2] J. Lindenstrauss and L. Tzafriri, *Classical Banach spaces I, sequence spaces*, *Ergeb. Math.* 92, Springer-Verlag 1977.

Aude Dalet

Let M be a pointed metric space and $Lip_0(M)$ the space of Lipschitz functions vanishing at 0. Endowed with the Lipschitz norm, this space is a Banach space. Its unit ball being compact for the pointwise topology, it is a dual space. Let us call its canonical pre-dual the Lipschitz-free space over M .

Despite the simplicity of their definition very little is known about the linear structure of Lipschitz-free spaces. In this talk we will study the Lipschitz-free space over a proper ultrametric space and prove that it is isomorphic to the dual of a space which is isomorphic to c_0 , in particular this Lipschitz-free space has the metric approximation property.

MINIMAL PROJECTIONS ONTO HYPERPLANES IN FINITE DIMENSIONAL
NORMED SPACE

Tomasz Kobos

The *relative projection constant* $\lambda(Y, X)$ of normed spaces $Y \subset X$ is defined as $\lambda(Y, X) = \inf\{\|P\| : P \in \mathcal{P}(X, Y)\}$, where $\mathcal{P}(X, Y)$ denotes the set of all continuous projections from X onto Y . By the well-known result of Bohnenblust for every n -dimensional normed space X and its subspace Y of codimension 1 the inequality $\lambda(Y, X) \leq 2 - \frac{2}{n}$ holds. This bound is optimal as the examples of $X = \ell_1^n$ or $X = \ell_\infty^n$ show. The aim of the talk is to study the situation in which the relative projection constant is maximal possible or close to the maximal. We derive an equivalent condition for the equality $\lambda(Y, X) = 2 - \frac{2}{n}$. In the three dimensional case it is enough to obtain a complete description of the equality case. Moreover, by using a stability version of this condition we are able to prove that every three-dimensional space has a subspace with the projection constant less than $\frac{4}{3} - 0.007$. This gives a non-trivial upper bound in the problem posed by Bosznay and Garay. In the general case, we can upper bound the number of $(n - 1)$ -dimensional subspaces of an n -dimensional normed space with the maximal relative projection constant. As a consequence, every n -dimensional normed space has an $(n - 1)$ -dimensional subspace with the projection constant strictly less than $2 - \frac{2}{n}$.

EXTREME DIFFERENCES ON THE SIZE OF WEAKLY OPEN SUBSETS

Ginés López-Pérez

A Banach space is said to have the slice diameter two property (slice-D2P), respectively diameter two property (D2P), or strong diameter two property (SD2P), if every slice, respectively nonempty relatively weakly open subset, or convex combination of slices, in its unit ball has diameter 2. The above diameter two properties have strong relations with Daugavet property or renorming with an octahedral norm. The aim of this talk is showing the extreme differences unknown up to now between the above diameter two properties and how finding out Banach spaces satisfying slice-D2P and failing D2P in an extreme way and also Banach spaces satisfying D2P and failing SD2P in an extreme way. (Joint to Julio Becerra Guerrero and Abraham Rueda Zoca)

GROUP APPROXIMATION IN CAYLEY TOPOLOGY AND COARSE GEOMETRY:
FIBERED COARSE EMBEDDING

Masato Mimura

The notion of FCEs (*Fibered Coarse Embeddings*) is introduced by Chen, Q. Wang, and Yu. Roughly speaking, an FCE is a “local form” of a CE (Coarse Embedding). They show that for a “nice” space, the FC embeddability into a Hilbert space implies the maximal coarse Baum–Connes conjecture; and that into an Hadamard manifold implies the coarse Novikov conjecture. Chen, Q. Wang, and X. Wang show that a box space admits FCEs into a Hilbert space iff the mother group has a-T-menability (a.k.a. the Haagerup property). In particular, the Selberg expander *does* admit an FCE into a Hilbert space. In this talk, we extend this theorem of Chen–Wang–Wang by providing a general framework to study coarse disjoint union of finite groups. To do this, we employ the concept of *the space of marked groups* of R. I. Grigorchuk. Here are two applications of our results: 1) We give the first (and explicit) example of expanders with the following properties: it does not admit FCEs into any “non-singular” CAT(0) spaces, such as, Hilbert spaces, Hadamard manifolds, and Euclidean buildings associated with linear groups; but it does admit a *bi-Lipschitz* embedding into a (“singular”) CAT(0) space. 2) We answer the question, raised by Q. Wang, X. Wang, and Yu, of whether “the FC embeddability into a Hilbert space” is stable under (finite) direct products. This talk is based on part II of our joint project with H. Sako (Niigata, Japan).

n-TUPLED COINCIDENCE POINT THEOREMS IN PROBABILISTIC MENGER
SPACES

Penumarthy Parvateesam Murthy

In this paper, the new concepts of *n*-tupled coincidence point for pair of mappings $T : X^n \rightarrow X$ and $A : X \rightarrow X$ are introduced in Menger Spaces. Utilizing the properties of the pseudo-metric and the triangular norm, we establish *n*-tupled coincidence point results using weak compatibility of functions as well as *n*-tupled fixed point theorems for several hybrid probabilistic ψ -contractions with a gauge function in Menger Spaces. Our main results are also general and results do not require the conditions of continuity and monotonicity for ψ . Our theorems generalizes the results of J. Jachymski, Xiao, Zhu and Cao, and Imb-dad, Soliman, Choudhary and Das. At the end an example is given to support our main theorem.

APPROXIMATION PROPERTIES IN LIPSCHITZ-FREE SPACES

Eva Pernecká

For a metric space M equipped with a distinguished element 0 , the Lipschitz-free space $\mathcal{F}(M)$ is a predual of the space of all real-valued Lipschitz functions on M vanishing at 0 . We will discuss approximation properties of Lipschitz-free spaces. We will present recent results saying that $\mathcal{F}(M)$ has the bounded approximation property for doubling metric spaces M , the metric approximation property for certain subsets M of \mathbb{R}^N with respect to any norm on \mathbb{R}^N , and a monotone finite-dimensional Schauder decomposition and a Schauder basis for ℓ_1^N and ℓ_1 . The talk will be based on joint work with P. Hájek, G. Lancien and R. Smith.

COMPACT CONVEX SETS ADMITTING A STRICTLY CONVEX FUNCTION

Matias Raja

In the frame of a locally convex space X , we will consider the class $SC(X)$ of convex compact subsets $A \subset X$ such that there exists $f : A \rightarrow \mathbb{R}$ which is lower semicontinuous and strictly convex. This class has some links with some other class of compacta studied in topology, as the descriptive or the fragmentable. We will give an embedding result for those compacta into strictly convex duals. Finally, we will discuss the existence of exposed points and related notions. This is part of a joint work with L.C. García Lirola and J. Orihuela.

THE FACES-RADON-NIKODÝM PROPERTY IN BANACH SPACES

David Salas

We introduce a Radon-Nikodým-like property related to exposed faces instead to exposed points of the closed convex bounded sets: namely, the FRNP. Some properties and characterizations of the class of spaces with the FRNP are given. The main questions about this property are 1) Is it a 3-space property? and 2) Is it equivalent or not to the RNP?

SOME BEST PROXIMITY POINT THEOREMS IN G - METRIC SPACES

Deepak Singh

When a non-self mapping has no fixed points, it could be interesting to study the existence and uniqueness of some points that minimize the distance between an origin and its corresponding image. These points are known as *best proximity points* and they were introduced by Fan [*Mathematische Zeitschrift*, vol. 112, no. 3 pp. 234-240, 1969]. Interestingly, best proximity point theorems also serve as a natural generalization of fixed point theorems, a best proximity point becomes a fixed point if the mapping under consideration is a self-mapping. Study of this kind of points and their properties has become one of the newest branches of fixed point theory, and many interesting results, generalizing the notion of fixed point, have been presented.

Very recently, N. Hussain et al. [*Abstract and Applied Analysis*, vol. 2014, art. ID 837943] introduced certain new class of proximal contraction mappings and established the best proximity point theorem in G - metric spaces. In this note, acknowledging the aforesaid concept, some best proximity point theorems are proved under generalized cyclic contraction condition which is new for this setting in the frame work of G - metric spaces.

Suitable examples are also presented which substantiate the genuineness of our investigations in this note.

COARSE QUOTIENT MAPPINGS BETWEEN METRIC SPACES

Sheng Zhang

We will introduce the definition of coarse quotient mapping and show that several results for uniform quotient mapping also hold in the coarse setting. In particular, the following results will be presented: (i) Any Banach space that is a coarse quotient of $L_p \equiv L_p[0, 1]$, $1 < p < \infty$, is isomorphic to a linear quotient of L_p ; (ii) ℓ_q is not a coarse quotient of ℓ_p for $1 < p < q < \infty$.

4 Posters

MOORE-PENROSE INVERSE AND NORMAL OPERATORS

Safa Menkad