Banach spaces and optimization: Conference on the occasion of Robert Deville's 60th birthday

Métabief, June 16–21, 2019.

Plenary talks

Daniel Azagra (Universidad Complutense Madrid)

Locally $C^{1,1}$ convex extensions of jets and Lusin properties of convex functions.

Let E be an arbitrary subset of $\mathbb{R}^n$, and $f : E \to \mathbb{R}$, $G : E \to \mathbb{R}^n$ be given functions. We provide necessary and sufficient conditions for the existence of a convex function $F \in C^{1,1}_{\text{loc}}(\mathbb{R}^n)$ such that $F = f$ and $\nabla F = G$ on E. We give a useful explicit formula for such an extension F. We also present an application of our result, concerning a $C^{1,1}_{\text{loc}}$ property of convex functions. More precisely, if a convex function $f : \mathbb{R}^n \to \mathbb{R}$ is not of class $C^{1,1}_{\text{loc}}$, then f is essentially coercive (in the sense that $\lim_{|x|\to\infty} f(x) - \ell(x) = \infty$ for some linear function ℓ) if and only if for every $\varepsilon > 0$ there exists a convex function $g : \mathbb{R}^n \to \mathbb{R}$ such that g is of class $C^{1,1}_{\text{loc}}$ and $\mathcal{L}^n (\{x \in \mathbb{R}^n : f(x) \neq g(x)\}) < \varepsilon$, where $\mathcal{L}^n$ denotes Lebesgue's measure). This application is a joint result with Piotr Hajlasz.

Aris Daniilidis (Universidad de Chile)

Spaceability of the space of Clarke-saturated Lipschitz functions.

We show that the set of Lipschitz continuous functions that saturate their Clarke subdifferential at every point of their domain contains a linear subspace of uncountable dimension. This result goes in the line of previous results of J.Borwein, W. Moors and X. Wang, delimiting the utility of Clarke subdifferential to special subclasses of Lipschitz functions. We shall comment on these latter classes and show that it is still possible to use Clarke criticality and expect Sard type results even beyond the semialgebraic case, by means of a regularity based on cardinality.

Based on a recent joint work with Gonzalo Flores (University of Chile).

Gilles Godefroy (CNRS, Université Pierre et Marie Curie)

Banach spaces of continuous functions on compact sets.

The linear structure of Banach spaces of the form $C(K)$ is well-understood when K is metrizable, but not so well when it is not. Moreover, the Lipschitz classification of $C(K)$ Banach spaces is still quite mysterious even when K is metrizable. We will discuss old and new results in this field of research, related with Robert Deville's contribution to the domain.

Antonio Guirao (Universitat Politècnica de València)

Remarks on the set of norm-attaining functionals and differentiability.

For a general Banach space X, the structure of the set of norm attaining functionals (usually denoted by $NA(X)$) is still far from being completely understood. Celebrated results by the community are: that $NA(X) = X^*$ if and only if X is reflexive (James' Theorem) and that $NA(X)$ is norm-dense in X^* (Bishop-Phelps' Theorem).

For non-reflexive Banach spaces still few can be said in general about $NA(X)$ beyond its norm-density. In our research, we have found a *more constructive* version of Bishop-Phelps-Bollobás Theorem in WCG spaces by applying the also celebrated Deville-Godefroy-Zizler smooth variational principle (by means of a renorming trick). Our second aim is to establish in the separable case a parallelism between the residuality of $NA(X)$ in X^* and the density of Fréchet differentiable points of the dual norm, obtaining for the separable case some characterizations of RNP and CPCP.

This is a joint work with V. Montesinos and V. Zizler

Petr Hájek (Czech Technical University and Czech Academy of Sciences)

WCG spaces without a norming M-basis.

We outline the construction of a WCG (in fact Hilbert generated) space which has no norming M-basis. This solves an old problem of John and Zizler from 1974.

Jesús Jaramillo (Universidad Complutense Madrid)

Invertibility of nonsmooth maps between Banach spaces.

We consider in this talk the invertibility of nonsmooth mappings between infinite-dimensional Banach spaces. We first survey some classical inversion results in the smooth setting. Then we introduce, as a main tool, an analogue of the notion of pseudo-Jacobian of Jeyakumar and Luc in the infinite-dimensional setting. Using this, we obtain several local and global inversion results. In particular, we give a version of the classical Hadamard integral condition in this context.

Sebastián Lajara (Universidad de Castilla-La Mancha)

Norming subspaces of Banach spaces.

In this talk, we present a characterization of the property that a closed subspace Z of the dual of a Banach space E is norming for a closed subspace X of E. This result is used to prove that, if Z is w^*-closed or X is reflexive, the pair of conditions X *is total over* Z and Z *is norming for* X (as well as Z *is total over* X and X *is norming for* Z), entail that $E = X \oplus Z_\perp$. Some applications to the study of norming bibasic systems in Banach spaces are also given.

The talk is based on a joint work with V. P. Fonf, S. Troyanski and C. Zanco.

Etienne Matheron (Université d'Artois)

Subseries convergence and the possibility of laziness.

In this talk, I will discuss a lemma concerning subseries convergence in topological groups. Originally, this lemma was isolated as a tool to study "strong sequential continuity", a notion intermediate between continuity and uniform continuity. However, it turns out that the lemma can also be used to give surprisingly short and almost identical proofs of several basic results in functional analysis.This is a very old work, partly joint with Alexander Borichev and Robert Deville.

Julian Revalski (Bulgarian Academy of Sciences)

Variational principles for supinf problems.

We propose variational principles for supinf problems for functions of two variables. Conditions are given which assure that the objective function can be perturbed by continuous functions with arbitrarily small norms in such a way that the supinf problem for the perturbed function has a solution. The notion of well-posedness for such problems is also investigated.

Short talks

Ramón Aliaga (Universitat Politècnica de València)

Supports in Lipschitz-free spaces and applications to extremal structure.

Some new results on the geometry of Lipschitz-free spaces over general metric spaces will be presented. It will be shown how to define a general notion of support for elements of Lipschitz-free spaces, and how these can be weighted with Lipschitz functions of bounded support. As an application of these techniques, we will show how to characterize several types of extreme points of the unit ball involving elementary molecules and positive elements. The results will be based on joint efforts with E. Pernecká, C. Petitjean and A. Procházka.

Bruno de Mendonça Braga (York University)

Nonlinear geometry of operator spaces.

The study of the nonlinear geometry of Banach spaces is currently a very active topic of research. In this talk, I will translate several notions of nonlinear embeddings/equivalences between Banach spaces to the operator spaces scenario. With those notions in hand, I will discuss some results that have analogs for operator spaces and give applications of those to the theory of operator spaces. (This is an ongoing project with Thomas Sinclair.)

Jesús M. F. Castillo (Universidad de Extremadura)

Twisted Hilbert spaces, explained by themselves.

A Banach space X is called a twisted Hilbert space if it contains a Hilbert subspace H so that X/H is also a Hilbert space. In a sense, they are the closest thing to Hilbert spaces ... without being Hilbert if such object exist. And they do: the Kalton-Peck space Z_2 [9] is *the* archetypal twisted Hilbert (non Hilbert) space, although not the first one in order of appearance, which was the space ELP constructed by Enflo, Lindenstrauss and Pisier [8]. But, while the ELP space is the outcome of an existence result, the Kalton-Peck space has deep connections with different parts of mathematics [10, 11].

This talk is devoted to display both its known and its conjectured properties [4], to exhibit new relevant twisted Hilbert spaces appearing in nature [5, 6, 12], to explain why Z_2 is at the core of the whole theory of twisted sums and why, in the same way that there is one Hilbert space in each subcategory of Banach spaces, there also seems to be one Kalton-Peck space [1, 3, 7] in each subcategory.

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Sheldon Dantas (Czech Technical University in Prague)

Smooth norms on dense subspaces.

It was recently asked by Antonio Guirao, Vicente Montesinos, and Václav Zizler whether the normed space $\mathcal{F}$ of all finitely supported vectors in $\ell_1(\Gamma)$, endowed with the ℓ_1-norm, admits a Fréchet smooth norm, where Γ is an uncountable set. In this talk, we present some results in this direction and we also show that, for certain Banach spaces X, there exists a dense subspace Y which admits an analytic norm and it can be chosen to approximate the original one.

This is a work in progress with Petr Hájek and Tommaso Russo.

Noé de Rancourt (Université Paris Diderot)

Hilbert-avoiding dichotomies and generalizations of the homogeneous space problem.

The solution, by Gowers and Komorowski–Tomczak-Jaegermann, of the homogeneous space problem, raised the question of the number of subspaces that a separable, non-Hilbertian Banach space can have, up to isomorphism. In particular, Ferenczi and Rosendal conjectured that such a space should have continuum-many pairwise non-isomorphic subspaces.

In this talk, I will present two Banach-space dichotomies that could help for this conjecture, and in particular for proving that, if counterexamples exist, then some of them must have an unconditional basis. The first dichotomy is very similar to Gowers' first dichotomy between HI spaces and spaces with an unconditional basis, but is Hilbert-avoiding, that is, it ensures that the obtained subspace is non-Hilbertian. The statement of this dichotomy involves a new class of spaces, hereditarily Hilbert-primary (HHP) spaces, that are spaces containing no topological direct sum of two non-Hilbertian subspaces. The second dichotomy is, in the same way, a Hilbert-avoiding version of Ferenczi–Rosendal's dichotomy between minimal spaces and tight spaces. These two dichotomies, combined, enable to reduce the question of the existence of counterexamples with an unconditional basis to the forementioned conjecture, to questions about the structure of subspaces of HHP spaces.

Olav G. Dovland (University of Agder, Norway)

Subspaces of $C(K)$ where some Urysohn's lemma still works.

Our starting point is Urysohn's lemma which we can state as *whenever* x_0 *is a point in a compact Hausdorff space* L*, and* U *is an open set containing* x_0*, then there is an* $f \in C_0(L)$ *such that* $f(x_0) = 1$ *but* $f \equiv 0$ *off* U. For uniform algebras we have something a bit weaker, but still much stronger than the following

Definition 1. A linear subspace $\mathcal{A}$ of $C_0(L)$ is *somewhat regular* if, whenever V is a non-empty open subset of L and $0 < \varepsilon < 1$, there is an $f \in \mathcal{A}$ such that

$$\|f\| = 1 \quad \text{and} \quad |f(x)| \leq \varepsilon \text{ for every } x \in L \setminus V.$$

Such spaces have some structure that we explore. Our main results are:

Theorem 2. *A somewhat regular subspace of* $C_0(L)$

- *has the symmetric strong diameter 2 property*
- *is almost square whenever* L *is non-compact*
- *has the Daugavet property whenever* L *does not contain isolated points*
- *contains an* ε*-isometric copy of* c_0 *whenever* $0 < \varepsilon < 1$

All needed definitions will be given during the talk that we will do our best to make understandable and enjoyable for anyone that would like to listen.

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Michael Dymond (Universität Innsbruck)

Differentiability of typical Lipschitz functions.

We consider the question of whether Rademacher's theorem on almost everywhere differentiability of Lipschitz functions admits a refinement for typical Lipschitz functions. In dimension one, Preiss and Tišer prove that the typical Lipschitz function is significantly better differentiable than Rademacher's theorem suggests. We discuss the situation in higher dimensions. Moreover, we present a recent example of an exceptionally tiny subset of the plane in which the typical Lipschitz function has many differentiability points. The research presented is in part joint work with Olga Maleva.

Jean Esterle (Université de Bordeaux)

On some extensions to Banach spaces of the notion of Hilbert-Schmidt operator.

Let E and F be real Banach spaces. A bounded linear operator $u : E \to F$ is said to be a *pre Hilbert-Schmidt* operator if $w \circ u \circ v : H_1 \to H_2$ is Hilbert Schmidt for every bounded operator $v : H_1 \to E$ and every bounded operator $w : F \to H_2$, where H_1 and H_2 are Hilbert spaces. Also a Banach space G is said to be a Hilbert-Schmidt space if $u \circ v$ is Hilbert-Schmidt for every bounded operator $u : G \to H_2$ and every bounded operator $v : H_1 \to G$ where H_1 and H_2 are Hilbert spaces.

The class $PS_2(E, F)$ of pre Hilbert-Schmidt operators equals the class $\gamma(E, F)$ of radonifying operators from E into F if F has type 2. We conjecture that in general $u \in PS_2(E, F)$ if and only if u factors through some Hilbert-Schmidt space G (this condition is obviously sufficient).

Joint work with S.A. Abdillah and B. Haak

Luis Carlos García Lirola (Kent State University)

On strongly norm attaining Lipschitz maps.

We study the set $SNA(M, Y)$ of those Lipschitz maps from a complete metric space M to a Banach space Y which strongly attain their Lipschitz norm (i.e. the supremum defining the Lipschitz norm is a maximum). Extending previous results, we prove that this set is not norm-dense when M is a length space, when M is a closed subset of $\mathbb{R}$ with positive Lebesgue measure, and when M is the unit circle. This provides new examples which have very different topological properties than the previously known ones. On the other hand, we study the linear properties which are sufficient to get Lindenstrauss property A for the Lipschitz-free space $\mathcal{F}(M)$ over M, and show that all of them actually provide the norm-density of $SNA(M, Y)$. Finally, we show that $SNA(M, \mathbb{R})$ is weakly sequentially dense in the space of all Lipschitz functions for all metric spaces M.

This is part of a joint work with B. Cascales, R. Chiclana, M. Martín and A. Rueda Zoca.

Rainis Haller (University of Tartu)

A metric characterization of the symmetric strong diameter two property in Lipschitz spaces.

We show that, for Lipschitz function spaces, the weak* strong diameter two property (meaning that finite convex combinations of weak* slices of the unit ball have diameter 2) is different from the weak* symmetric strong diameter two property (meaning that, given a finite number of weak* slices of the unit ball, there exists a direction such that all these slices contain a line segment in this direction of length "almost" 2). A characterization of the second-named property of the Lipschitz space in terms of the corresponding metric space is given. A similar characterization for the first-named property was obtained by A. Procházka and A. Rueda Zoca.

This research is a part of the PhD thesis project of Andre Ostrak supervised by R. Haller and M. Põldvere at University of Tartu.

Tomasz Kochanek (University of Warsaw)

Approximately order zero maps between C*-algebras.

We deal with an Ulam-type stability problem whether an approximately zero-product-preserving (order zero) map between C*-algebras must be close to a zero-product-preserving map. More precisely, for given C*-algebras A, B and a symmetric, bounded, linear operator $\varphi\colon A \to B$ satisfying $\|\varphi(x)\varphi(y)\| \leq \varepsilon\|x\|\|y\|$ for all self-adjoint $x, y \in A$ with $xy = 0$, we ask whether there exists a zero-product-preserving operator $\psi\colon A \to B$ with $\|\varphi - \psi\| \leq \delta(\varepsilon)$, where $\delta(\varepsilon) \to 0$ as $\varepsilon \to 0$. During the talk, I shall sketch a proof that such a stability effect holds true in the case where A is nuclear (amenable as a Banach algebra) and B is a von Neumann algebra.

Eva Kopecká (University of Innsbruck)

Alternating projections converge on a dense subspace reasonably fast.

Let $L_1, L_2, \ldots, L_K$ be a family of K closed subspaces of a Hilbert space H. If $\bigcap_{i=1}^{\infty} L_i = \{0\}$, then $Y = L_1^{\perp} + \cdots + L_K^{\perp}$ is dense in H. We show that there is $\alpha > 0$ such that if $y \in Y$, then $\|T^n(y)\| = o(n^{-\alpha})$, thus proving a conjecture of Deutsch and Hundal. Here $T = P_K \ldots P_1$ stands for the cyclic product of the orthogonal projections $P_1, P_2, \ldots, P_K$ onto the subspaces $L_1, L_2, \ldots, L_K$, respectively.

It is well known that various convergence properties of the infinite products of the P_i's depend on Y being closed. If Y is not closed, products of projections might diverge. The cyclic products $\{T^n(z)\}_{n=1}^{\infty}$ converge for every $z \in H$, but there is a dichotomy. The convergence is fast if and only if Y is closed; otherwise the convergence is as slow as one likes for appropriately chosen initial vectors z. Thus Y provides in any case a dense subspace of starting points for which the convergence of cyclic products is reasonably fast.
(Joint work with P. Borodin)

Ondřej Kurka (Czech Academy of Sciences)

The isomorphism class of c_0 is not Borel.

We show that the class of all Banach spaces which are isomorphic to c_0 is not Borel with respect to the Effros Borel structure of separable Banach spaces. This answers a question posed by G. Godefroy.

To prove this result, we modify a construction due to S. A. Argyros, I. Gasparis and P. Motakis, which is performed by the famous Bourgain-Delbaen method. At the same time, we involve the Tsirelson-like spaces of S. A. Argyros and I. Deliyanni. In this way, we construct a $\mathcal{L}_\infty$-space $\mathfrak{X}_{\mathcal{M}}$, with a compact system $\mathcal{M}$ of sets of natural numbers as a parameter. The gist of this construction is that $\mathfrak{X}_{\mathcal{M}}$ is isomorphic to c_0 if and only if $\mathcal{M}$ contains an infinite set.

In fact, using this construction, it is not difficult to show that the same result holds for the space $C(K)$ whenever K is an infinite compact metrizable space.

We also obtain for instance that the class of spaces with an unconditional basis is not Borel.

Johann Langemets (University of Tartu)

Bidual octahedral renormings and strong regularity in Banach spaces.

We prove that every separable Banach space containing ℓ_1 can be equivalently renormed so that its bidual space is octahedral, which answers, in the separable case, a question by Godefroy from 1989. As a direct consequence, we obtain that every dual Banach space, with a separable predual, failing to be strongly regular (that is, without convex combinations of slices with diameter arbitrarily small for some closed, convex and bounded subset) can be equivalently renormed with a dual norm to satisfy the strong diameter two property (that is, such that every convex combination of slices in its unit ball has diameter two).

The talk is based on a joint work with Prof. Ginés López.

Vegard Lima (University of Agder)

Banach spaces where convex combinations of slices are relatively weakly open.

We give examples of Banach spaces where every finite convex combination of slices of the unit ball is relatively weakly open in the unit ball.

The proofs heavily use that in some Banach space taking convex combinations of points in the unit ball is an open map. We say that the space has property (co).

For finite dimensional space property (co) implies that convex combination of slices are relatively weakly open. V. Kadets has proved the other direction.

We will also mention results by G. López-Pérez, M. Martín, and A. Rueda Zoca giving connections to diameter two properties.

This talk is based on joint work with Trond Arnold Abrahamsen, Julio Becerra Guerrero, Rainis Haller, and Märt Põldvere.

André Martiny (University in Agder)

On octahedrality and Müntz spaces.

We discuss how every Müntz space can be written as a direct sum of Banach spaces X and Y, where Y is almost isometric to a subspace of c and X is finite dimensional. We apply this to show that no Müntz space is locally octahedral or almost square.

Carlos Mudarra (ICMAT)

Smooth convex extensions of jets from convex subsets of $\mathbb{R}^n$.

In this talk we discuss the existence of a Whitney-type extension theorem for convex functions of class $C^m(\mathbb{R}^n)$, where $m \geq 2$. More precisely, given a closed convex subset E of $\mathbb{R}^n$ and a m-jet $\{P_x^m\}_{x\in E}$ defined on E, we look for necessary and sufficient conditions on $\{P_x^m\}_{x\in E}$ for the existence of a convex function $F \in C^m(\mathbb{R}^n)$ such that the m-jet $J_x^m F$ of F at the point x coincide with P_x^m for every $x \in E$. We provide a full solution to this problem when $m = \infty$ and E is a compact convex subset of $\mathbb{R}^n$. This is a joint work with Daniel Azagra.

Rihhard Nadel (University of Tartu)

Symmetric strong diameter two property.

The talk is based on the joint paper [1]. We only look at non-trivial real Banach spaces. A slice of B_X is a set of the form

$$S(x^*, \alpha) = \{x \in B_X \mid x^*(x) > 1 - \alpha\},$$

where $x^* \in S_{X^*}$ and $\alpha > 0$.

Definition 1 ([2])**.** A Banach space is said to have the *strong diameter 2 proprty* (SD2P) if every convex combination of slices of the unit ball has diameter equal to 2.

We investigate the following property, which first appeared in [2], but was singled out and studied in [3].

Definition 2. A Banach space X has the *symmetric strong diameter 2 property* if for every finite family $\{S_i\}_{i=1}^n$ of slices of B_X and $\varepsilon > 0$ there exist $x_i \in S_i$ and $y \in B_X$, independently of i, such that $x_i \pm y \in S_i$ and $\|y\| > 1 - \varepsilon$.

We'll show how the SSD2P relates to the closely related and widely studied strong diameter 2 property (SD2P) and the property almost squareness (ASQ), among others.

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Hong Wai Ng (Nanyang Technological University)

On ordinal ranks of Baire class functions.

The theory of ordinal ranks on Baire class 1 functions developed by Kechris and Loveau was recently extended by Elekes, Kiss and Vidnyánszky to Baire class ξ functions for any countable ordinal $\xi \geq 1$. In this talk, we will answer two of the questions raised by them in their paper entitled 'Ranks on the Baire Class ξ functions' published in Trans. Amer. Math. Soc. 2016.

This is a joint work with Denny H. Leung of National University of Singapore and Wee-Kee Tang of Nanyang Technological University.

Chadi Nour (Lebanese American University)

A general result about inner regularization of sets.

In the paper [4], Nour, Saoud and Takche provided, for a given closed set $S \subset \mathbb{R}^n$, an inner approximation of S by sets satisfying the *interior sphere condition*. The fact that their approximation sets satisfy the interior sphere condition with variable radius, allows them to approach any corner and to use the Pompeiu-Hausdroff convergence even if the set S is unbounded. To guarantee the existence of such approximation, Nour, Saoud and Takche assumed the nonemptiness of the interior of S^*, the star of S, (see [4, Theorem 3.1]). This assumption can be seen as the following: There is a fixed closed ball B_0 in S such that each point in S is the vertex of a circular convex cone in S having B_0 as base. Nour, Saoud and Takche generalized this result in [4, Theorem 3.3] where the closed balls bases are allowed to be different but are moving tangentially on the boundary of a set $A \subset S$. The existence of such balls is guaranteed by the φ_0-*convexity* (or *proximal smoothness*) assumption of A.

The goal of this talk is to present a generalization of the main result of [4] to cover more sets S. The main idea is to replace the circular convex cones used in [4] by *curved* ones. More precisely, we will replace the existence of the φ_0-convex set A with the properties mentioned above by the existence of a family of curves that cover the set S in a special geometric and topological way. Our new assumptions will be more general compared to the assumptions of [4].

Our new results, together with the results of [4], relate to results of Clarke, Ledyayev and Stern [1] and Cornet and Czarnecki [2, 3]. But they differ from these results by several features. The main ones are that we do not make a *wedged* (or *epi-Lipschitz*) assumption on S and our approximation sets satisfy an interior sphere condition which is a different regularity property from the φ_0-convexity, used in [1, 2, 3], as is already shown in Nour, Stern and Takche [5].

This is a joint work with J. Takche.

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José Orihuela (Universidad de Murcia)

Renormings through Deville's master lemma.

Tanmoy Paul (Indian Institute of Technology Hyderabad)

On strongly Hahn-Banach smooth subspaces of Banach spaces.

A closed subspace Y of a Banach space X is said to be Strongly Hahn-Banach smooth subspace if every $f \in Y^*$ posses a unique norm preserving extension to the whole of X and there is a linear projection $P : X^* \to Y^\perp$ such that $\|Px^*\| < \|x^*\|$ (if $Px^* \neq x^*$). M-ideals are natural examples of strongly Hahn-Banach smooth subspaces but the class of strongly Hahn-Banach smooth subspaces is much bigger than the class of M-ideals. We discuss some transitivity properties of strongly Hahn-Banach smoothness, show that this property is separably determined and is stable with respect to discrete c_0 sums.

Eva Pernecká (Czech Technical University in Prague)

The notion of support for elements of Lipschitz-free spaces.

We will show that the class of Lipschitz-free spaces over closed subspaces of a complete metric space is closed under arbitrary intersections, and that this leads to a natural definition of *support* for elements of Lipschitz-free spaces. We will discuss some characterizations and properties of supports. Finally, we will have a look at the elements of Lipschitz-free spaces induced by Radon measures. The talk will be based on a joint work with Ramón J. Aliaga.

Yoël Perreau (Université Bourgogne Franche-Comté)

On the embeddability of the family of countably branching trees into quasi-reflexive Banach spaces.

Equipped with the standard hyperbolic distance, the set $T_N = \{\emptyset\} \cup \bigcup_{n=1}^N \mathbb{N}^n$ is called countably branching tree with N steps. In this talk, we look at the equi-Lipschitz embeddability of the family $(T_N)_{N \geq 1}$ into Banach spaces. We give a complete characterization of this property in terms of Szlenk index in the quasi-reflexive setting. This result is an asymptotic analogue of Bourgain's characterization of super-reflexivity in terms of (non)-equi-Lipschitz embeddability of the family of dyadic trees.

Katriin Pirk (University of Tartu)

Delta- and Daugavet-points in direct sums of Banach spaces.

We consider real infinite-dimensional Banach spaces. For a Banach space X, we say that $x \in S_X$ is a

(i) Δ-*point* if $x \in \overline{\operatorname{conv}}\, \Delta_\varepsilon(x)$ for every $\varepsilon > 0$,
(ii) *Daugavet-point* if $B_X \subset \overline{\operatorname{conv}}\, \Delta_\varepsilon(x)$ for every $\varepsilon > 0$,

where

$$\Delta_\varepsilon(x) = \{y \in B_X : \|x - y\| \geq 2 - \varepsilon\}.$$

It was shown in [1], on one hand, that if Banach spaces X and Y have Δ-points, then $X \oplus_N Y$ has Δ-points for any absolute normalized norm N. On the other hand, it was proved that for some norms N the direct sum $X \oplus_N Y$ cannot have any Daugavet-points, even though there exist norms N such that if X and Y have Daugavet-points, the direct sum $X \oplus_N Y$ also has Daugavet-points. However, in the direction from component spaces to direct sum there were some norms N for which the question was left open for Daugavet-points. In addition, there were no results in the opposite direction, from the direct sum to components, for neither Δ-points nor Daugavet-points. In recent collaboration with R. Haller and T. Veeorg we completed the research started in [1], giving answers to these questions.

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Daniele Puglisi (University of Catania)

Banach algebras of Calkin type.

The talk will concern the question of what types of unital algebras can occur as Calkin algebras. Jointly work with P. Motakis and A. Tolias.

Nadezhda Ribarska (Sofia University)

Sufficient conditions for tangential transversality.

We present a general sufficient condition for tangential transversality and obtain as corollaries a result about tangential transversality of jointly massive sets as well as an abstract theorem on tangential transversality of subsets of the cartesian product of two Banach spaces.

Joint work with Stoyan Apostolov and Mikhail Krastanov.

Tommaso Russo (Czech Technical University in Prague)

On densely isomorphic normed spaces.

In this talk, based on a joint project with Sheldon Dantas and Petr Hájek, we shall be interested in dense subspaces of a given normed space. We will give some results and examples aimed at understanding how ‘different’ two dense subspaces of a normed space can be. In the second part of the talk, we shall, more specifically, concentrate our attention to the study of systems of coordinates in such normed spaces.

Alberto Salguero Alarcón (Universidad de Extremadura)

Kalton meets Fourier.

The purpose of this talk is to show, by means of Fourier analysis, how the celebrated Kalton-Peck space Z_2 generates a twisted sum of L_1 and L_∞. Recall that a twisted sum of two Banach spaces Y and X is another space Z containing Y as a subspace in such a way that $Z/Y = X$. The Kalton-Peck space Z_2 is one of the main examples of twisted sums: it is a non-Hilbert twisted sum of two Hilbert spaces.

Richard Smith (University College Dublin)

A topological characterization of dual strict convexity in Asplund spaces.

We say that a topological space Z has $(*)$ if there is a sequence $(\mathscr{U}_j)_{j=0}^\infty$ of families of open subsets of Z, with the property that given $x, y \in Z$, there exists $j \in \mathbb{N}$ such that

(1) $\{x, y\} \cap \bigcup \mathscr{U}_j$ is non-empty, and
(2) $\{x, y\} \cap U$ is at most a singleton for all $U \in \mathscr{U}_j$.

This property was introduced by J. Orihuela, S. Troyanski and the author several years ago, in relation to strictly convex norms on Banach spaces.

We show that if X is an Asplund space, then it admits an equivalent norm having a strictly convex dual norm if and only if the dual unit sphere S_{X^*} (equivalently X^*), endowed with the w^*-topology, possesses $(*)$. It follows that this ostensibly geometric property of the space can in fact be characterized in purely non-linear, topological terms. This improves upon a previous characterization, obtained by the authors above, which required an additional linearity assumption.

Libor Veselý (University of Milan)

Intersection properties of the unit ball.

We say that a Banach space $(X, \|\cdot\|)$ has property (I) [property (GI)] if for each sequence [family] $\{\|\cdot\|_i\}$ of equivalent norms on X such that

$$\|\cdot\| = \sup_i \|\cdot\|_i$$

the same formula holds for the corresponding bidual norms. We give characterizations of the above properties (some of them in terms of the so-called intermediate envelope), and we study them in $C(K)$ spaces and some other classical Banach spaces. We also characterize reflexivity in terms of renormings satisfying these properties. (This is a joint work with Carlo A. De Bernardi.)

Elroy Zeekoei (North-West University)

Dunford-Pettis-type operators as ideal extensions.

The purpose of this talk is to introduce the notions of the so-called p-convergent and weak* p-convergent operators. We then define the so-called "left" and "right" ideal extendibility of operators in general, and describe the p-convergent and weak* p-convergent operators as ideal extensions of the compact operators and the p-convergent operators respectively.

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[3] Fourie, J.H., Zeekoei, E.D. Classes of sequentially limited operators. *Glasgow Mathematical Journal* **58**, 573–586 (2016)

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Nadya Zlateva (Sofia University)

Variational principle for integral functional.

We provide a general method for proving existence of solutions of suitable perturbations of certain variational problems. A novel variational principle enables perturbing only the integrand, thus preserving the form of the problem. Joint work with Milen Ivanov.

Posters

Mohammed Bachir (Université Paris 1)

Limited operators and differentiability.

We characterize the limited operators by differentiability of convex continuous functions. Given Banach spaces Y and X and a linear continuous operator T from Y into X, we prove that T is a limited operator if and only if, for every convex continuous function f from X into $\mathbb{R}$ and every point y of Y, $f \circ T$ is Fréchet differentiable at y whenever f is Gâteaux differentiable at $T(y)$.

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Miguel García Bravo (Universidad Autónoma de Madrid)

Extraction of critical points of smooth functions on Banach spaces.

Colin Petitjean (Université Bourgogne Franche-Comté)

Concentration Phenomena for Kalton's interlaced graphs.