# Diametral dimension(s) and prominent bounded sets

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## Definition

The *diametral dimension* of $E$ is

$$\Delta(E) := \left\{\xi \in \mathbb{C}^{\mathbb{N}_0} : \forall U \in \mathcal{U},\ \exists V \in \mathcal{U}, V \subseteq U,\ \text{s.t. } \xi_n \delta_n(V, U) \to 0\right\},$$

with $\delta_n(V, U) := \inf\{\delta > 0 : \exists L \subseteq E,\ \dim L \leq n,\ \text{s.t. } V \subseteq \delta U + L\}$.

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## NB

If $U$ is absolutely convex and absorbing, then $V$ is precompact with respect to $U$ iff $\delta_n(V, U) \to 0$.

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- $c_0 \subseteq \Delta(E)$.
- If $E$ is a l.c.s.,
  - $E$ is Schwartz $\Leftrightarrow c_0 \subsetneq \Delta(E) \Leftrightarrow \ell_\infty \subseteq \Delta(E)$;
  - $E$ is nuclear $\Leftrightarrow \left\{ \begin{array}{c} \exists p > 0 : \\ \forall p > 0, \end{array} \right\} (n^p)_n \in \Delta(E)$.

# Another diametral dimension...

## Definition

If $E$ is a t.v.s. and $\mathcal{U}$ is basis of 0-neighbourhoods,

$$\Delta_b(E) := \left\{\xi \in \mathbb{C}^{\mathbb{N}_0} : \forall B \text{ bounded}, \forall U \in \mathcal{U}, \xi_n\delta_n(B,U) \to 0\right\}.$$

**Reminder:** $\Delta(E) = \left\{\xi \in \mathbb{C}^{\mathbb{N}_0} : \forall U \in \mathcal{U}, \exists V \in \mathcal{U} \text{ s.t. } \xi_n\delta_n(V,U) \to 0\right\}$.

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## NB

$\Delta(E) \subseteq \Delta_b(E)$.

## Question (Mityagin)

$\Delta(E) = \Delta_b(E)$ for Fréchet spaces ?

# One observation

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Consequences

- If $E$ is not Montel: $\Delta(E) = \Delta_b(E) = c_0$.
- If $E$ is Montel but not Schwartz: $\Delta(E) = c_0 \subsetneq \Delta_b(E)$.

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$\rightsquigarrow$ New open question

$\Delta(E) = \Delta_b(E)$ for Fréchet-Schwartz spaces?

# One first result

## Notations

$$\Delta^\infty(E) := \left\{ \xi \in \mathbb{C}^{\mathbb{N}_0} : \forall U \in \mathcal{U},\ \exists V \in \mathcal{U},\ \text{ s.t. } \left(\xi_n \delta_n(V, U)\right)_n \in \ell_\infty \right\},$$

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**Theorem (2016, L.D., L. Frerick, J. Wengenroth)**

If $E$ is a Fréchet-Schwartz, then $\Delta^{\infty}(E) = \Delta_b^{\infty}(E)$.

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Theorem (2016, L.D., L. Frerick, J. Wengenroth)

If $E$ is a Fréchet-Schwartz, then $\Delta^{\infty}(E) = \Delta_b^{\infty}(E)$.
In particular, if $\Delta(E) = \Delta^{\infty}(E)$, then $\Delta(E) = \Delta_b(E)$.

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Which Fréchet-Schwartz spaces verify $\Delta(E) = \Delta^\infty(E)$?

- **Hilbertizable** Fréchet-Schwartz spaces and, in particular, **nuclear** Fréchet spaces (2016, L.D., L. Frerick, J. Wengenroth);
- **Köthe-Schwartz** sequence spaces (2017, F. Bastin, L.D.).

# Another idea...

We want

$$\Delta_b(E) = \left\{\xi \in \mathbb{C}^{\mathbb{N}_0} : \forall B \text{ bounded}, \forall U \in \mathcal{U}, \xi_n \delta_n(B, U) \to 0\right\}$$

$$\subseteq \Delta(E) = \left\{\xi \in \mathbb{C}^{\mathbb{N}_0} : \forall U \in \mathcal{U}, \exists V \in \mathcal{U} \text{ s.t. } \xi_n \delta_n(V, U) \to 0\right\}.$$

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$\rightsquigarrow$ Prominent bounded sets (2013, T. Terzioglu)

A bounded $B$ set of $E$ is *prominent* if $\forall U \in \mathcal{U}$, $\exists V \in \mathcal{U}$, $C > 0$ s.t. $\delta_n(V,U) \leq C\delta_n(B,V)$ $\forall n \in \mathbb{N}_0$.

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### Property

If $E$ has a prominent set, then $\Delta(E) = \Delta_b(E)$.

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### Property

If $E$ has a prominent set, then $\Delta(E) = \Delta_b(E)$.

### Warning!

The converse is false (e.g. for power series spaces of infinite type). (2013, T. Terzioglu)

# Looking for prominent sets...

## Reminder

A Fréchet space $E$, with a fundamental system of semi-norms $(\|.\|_n)_{n\in\mathbb{N}}$, has the *property* $(\overline{\Omega})$ if

$$\forall m \; \exists k \; \forall j \; \exists C > 0 : \left(\|x'\|_k^*\right)^2 \leq C\|x'\|_m^*\|x'\|_j^* \quad \forall x' \in E'$$

where $\|.\|_m^*$ is the dual norm of $\|.\|_m$.

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## Proposition (2017, F. Bastin, L.D. ; 2016, L.D., L. Frerick, J. Wengenroth)

If $E$ has the property $(\overline{\Omega})$, then it has a prominent bounded set. In particular, $\Delta(E) = \Delta_b(E)$.

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If $E$ has the property $(\overline{\Omega})$, then it has a prominent bounded set. In particular, $\Delta(E)=\Delta_b(E)$.

## Remarks (2016, L.D., L. Frerick, J. Wengenroth)

- For *regular Köthe spaces*, existence of prominent sets $\Leftrightarrow$ $(\overline{\Omega})$.
- There exist spaces with prominent sets but without $(\overline{\Omega})$ (e.g. $H(\mathbb{D})\times H(\mathbb{C})$).

Introduction: Diametral dimension

Some first results

Prominent bounded sets

Non-Fréchet spaces

# Counterexamples

## Main idea

If $E$ is a l.c.s. s.t. every bounded set is "finite-dimensional", then $\Delta_b(E) = \mathbb{C}^{\mathbb{N}_0}$.

**Reminders:** $\Delta_b(E) = \left\{\xi \in \mathbb{C}^{\mathbb{N}_0} : \forall B \text{ bounded}, \forall U \in \mathcal{U}, \xi_n \delta_n(B, U) \to 0\right\}$ and $\delta_n(B, U) = \inf \{\delta > 0 : \exists L \subseteq E,\ \dim L \leq n,\ \text{s.t. } B \subseteq \delta U + L\}$.

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## Purpose

Finding a family of Schwartz spaces $E$ with only finite-dimensional bounded sets s.t. $\Delta(E) \neq \mathbb{C}^{\mathbb{N}_0}$...

# Interesting properties

**Proposition** (1979, A. Wilansky)

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Proposition (1981, H. Jarchow)

If $E$ and $F$ are two l.c.s., $F$ **barrelled**, for which $\exists T : E \to F$ **linear, continuous, and surjective**, then $\Delta(E) \subseteq \Delta(F)$.

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If $E$ and $F$ are two l.c.s., $F$ **barrelled**, for which $\exists T : E \to F$ **linear, continuous, and surjective**, then $\Delta(E) \subseteq \Delta(F)$.
In particular, if $\mathcal{T}_1$ and $\mathcal{T}_2$ are 2 l.c.t. on a vector space $E$, with $\mathcal{T}_1$ barrelled and $\mathcal{T}_1 \leq \mathcal{T}_2$, then

$$\Delta(E, \mathcal{T}_2) \subseteq \Delta(E, \mathcal{T}_1).$$

# Counterexamples

**Theorem** (2017, F. Bastin, L.D.)

If $E$ is a vector space and

- $\mathcal{T}_1$ is a barrelled l.c.t. on $E$ s.t. $\Delta(E, \mathcal{T}_1) \subsetneq \mathbb{C}^{\mathbb{N}_0}$ (e.g. $E \equiv$ Köthe spaces),

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- $\mathcal{T}_2$ is a l.c.t. with only finite-dimensional bounded sets (e.g. $\sigma(E, E^*)$),
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$$\Delta(E, \mathcal{T}) \subsetneq \Delta_b(E, \mathcal{T}).$$

In particular, if $\mathcal{T}_1$ and $\mathcal{T}_2$ are Schwartz (resp. nuclear), then $\mathcal{T}$ is Schwartz (resp. nuclear).

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then

$$\Delta(E, \mathcal{T}) \subsetneq \Delta_b(E, \mathcal{T}).$$

In particular, if $\mathcal{T}_1$ and $\mathcal{T}_2$ are Schwartz (resp. nuclear), then $\mathcal{T}$ is Schwartz (resp. nuclear).

Indeed, $\Delta(E, \mathcal{T}) \subseteq \Delta(E, \mathcal{T}_1) \subsetneq \mathbb{C}^{\mathbb{N}_0} = \Delta_b(E, \mathcal{T})$.

Thank you for your attention!

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