Distortion of Lipschitz Functions on $c_0(\Gamma)$

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Introduction

Definition

Let X be a Banach space and $f : S_X \to \mathbb{R}$. We say f is **oscillation stable** if for every infinite dimensional subspace $Z \subset X$ and every $\varepsilon > 0$ there exists a subspace $Y \subset Z$ such that $|f(x) - f(y)| \leq \varepsilon$ for every $x,y \in S_Y$.

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Definition

A function $f : S_X \to \mathbb{R}$ is said to be **distorted** if there exists an $\varepsilon > 0$ such that for every infinite dimensional subspace Y of X there exist $x,y \in S_Y$ such that $|f(x) - f(y)| > \varepsilon$.

Distortion of Lipschitz mappings

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Theorem (Gowers)

Every Lipschitz function $f : S_{c_0} \to \mathbb{R}$ *is oscillation stable.*

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Theorem

There is a distorted Lipschitz function on ℓ_1.

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Let $(X, \|\cdot\|)$ be a Banach space with a symmetric (possibly uncountable) Schauder basis $\{e_\gamma\}_{\gamma\in\Gamma}$, where Γ is any nonempty set. We say that a function $f : X \to \mathbb{R}$ is **symmetric** if the value $f(x)$ is preserved under any permutation of the coordinates of x.

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Theorem (Hájek, N.)

There is a 1*-Lipschitz symmetric function* $F : S_{c_0(\Gamma)} \to \mathbb{R}$*, such that for every nonseparable subspace* $Y \subseteq c_0(\Gamma)$ *there are points* $x,y \in S_Y$ *such that* $|F(x) - F(y)| > \frac{1}{4}$.

Summary

There are even results on equivalent norms!

	Oscillation stable Lipschitz mappings	Oscillation stable norms
c_0	$\surd$	$\surd$
ℓ_1	X	$\surd$
ℓ_p	X	X
$c_0(\Gamma)$	X	$\surd$
$\ell_1(\Gamma)$	X	$\surd$
$\ell_p(\Gamma)$	X	X

Thank you for your attention.