List of abstracts

Xavier ANTOINE

Computational methods for the stationary state Gross-Pitaevskii equation

The aim of this talk is to present some numerical methods for computing stationary states of general Gross-Pitaevskii equations. The techniques are based on a conjugate normalized gradient flow formulation that is discretized through a well-adapted time scheme for stability reason. In addition, the spectral approximation in space is based on the combination of FFT approximation methods and preconditioned Krylov subspace solvers. Numerical examples will be treated based on the GPELab Matlab toolbox.

Sylvie BENZONI

Nonlinear waves in dispersive PDEs

The aim of the talk is to review some stability/instability results obtained with C. Mietka, P. Noble, and M. Rodrigues for periodic waves in a large class of dispersive PDEs, which includes the NonLinear Schrödinger equation, generalized Korteweg-de Vries equations, and dispersive perturbations of the Euler equations for compressible fluids.

Christophe BESSE

Exponential integrators for NLS equations with application to rotating BECs

This talk deals with the numerical integration in time of nonlinear Schrödinger equations. The main application is the numerical simulation of rotating Bose-Einstein condensates. Usually, one performs a change of unknown so that the rotation term disappears and one obtains as a result a nonautonomous nonlinear Schrödinger equation. We consider exponential integrators such as exponential Runge-Kutta methods and Lawson methods. We provide an analysis of the order of convergence and some preservation properties of these methods in a simplified setting. We present results with numerical experiments with realistic physical parameters.

Yanne CHEMBO

Advances in Kerr comb generation

We discuss some of the latest advances in the area of Kerr optical frequency combs. We provide an overview of the main theoretical and experimental results that we have obtained on this topic, and highlight the potential of Kerr combs for current or prospective applications. We finally discuss some of the open challenges that are to be met at the fundamental and applied level.

David CHIRON

Multiple branches of travelling waves in the 2D Gross-Pitaevskii model

In this talk, we will be interested in travelling waves solutions in the two-dimensional Gross-Pitaevskii equation, which is a Nonlinear Schrödinger equation with the constraint of modulus one at infinity. This equation possesses a branch of travelling waves which has been numerically obtained by Jones and Roberts for speeds between 0 and the speed of sound associated with acoustic waves. Some rigorous results justify the two extreme asymptotics : on the one hand for speeds close to 0, where we qualitatively observe vortices ; on the other hand for speeds close to the speed of sound, where the wave becomes a rarefaction pulse asymptotically described, after rescaling, by the Kadomtsev-Petviashvili I solitary wave. This branch may be obtained by minimizing the energy at fixed momentum. We shall present numerical results showing the existence of other branches of travelling waves, corresponding to the excited states. This is a joint work with Claire Scheid (Nice).

Aurélien COILLET

Exotic phase-locked states in Kerr frequency combs

Research on optical frequency combs generated in ring resonators has been extremely active in the past few years, with a large variety of experimental results and a very good understanding of the dynamics at stake in the resonator using the Lugiato-Lefever equation. We present new regimes of these Kerr frequency combs that we have not been able to completely explain. These combs show interesting properties: they all appear in the same small region of the parameter space, they are all “phase-locked”, and some of them present phase alignments with very well-defined pi-shifts . We have been able to use these states for time and frequency metrology applications, with the generation of an ultra-low phase noise microwave tone and by self-referencing the comb with a f-2f interferometer scheme.

Anne DE BOUARD

Stochastic models in fiber optics

We will review some mathematical and numerical results concerning the modeling of the interactions between nonlinearity, dispersion and noise in optical fibers. Two cases will be considered : the influence of inhomogeneities in dispersion managed fibers, and the random modeling of birefringence. Both cases lead to random nonlinear Schrödinger equations for which, on some specific scale, the noise is well approximated by a Gaussian white noise.

François GENOUD

Stable spatial solitons via bifurcation in inhomogeneous planar waveguides

I will start this talk by recalling (a derivation of) the paraxial approximation, yielding the nonlinear Schrödinger equation as the master equation for the propagation of spatial solitons in nonlinear waveguides. I will then present rigorous stability results based on bifurcation techniques in some inhomogeneous planar media.

Damia GOMILA

Noise fluctuations in Kerr frequency combs

Frequency combs (FCs) are of key importance in many high precision metrology applications. It has been recently shown that the formation of patterns and cavity solitons in ring microresonators is a valid strategy to generate high quality FCs. In this context Kerr combs are properly described by the Lugiato-Lefever equation (LLE). Noise broadens, however, the FC peaks ultimately limiting the precision of the devices. Here we present a study of the effects of the fluctuations in the LLE in the presence of noise. In particular we compute analytically the average mean squared amplitude of the fluctuations of a FC associate to a stationary periodic pattern and show how the base of the FC peaks broaden due to the excitation of soft (Eckhaus) modes. We compare the analytical results with numerical simulations of the full model and with some experimental data.

John JOST

Microresonator Optical Frequency Combs

The development of the optical frequency comb (OFC) based on femtosecond pulsed mode-locked lasers changed the field of frequency metrology. In the frequency domain, OFCs give an equidistant set of optical lines that can serve as a ruler for light. They can be used in a diverse array of applications from the most precise atomic clocks to spectroscopy. The focus of this talk will be on a new type of optical frequency comb created via parametric frequency conversion and the formation of dissipative temporal cavity solitons in an optical microresonator. The advent of this new type of OFC has opened up a new parameter space, enabling high repetition rates (typically >10 GHz) and broad spectral coverage from the near to the mid-infrared.

Christian KLEIN

Numerical study of 2+1 dimensional nonlinear dispersive PDEs

We present several numerical studies of solutions to PDEs from the family of nonlinear Schrödinger and Kortweg-de Vries equations. We study the formation of dispersive shocks and of potential blow-ups in the solutions.

Gregory KOZYREFF

Interaction between slow and fast scales in slowly varying envelope approximations

NLS, Lugiato-Lefever (LL) and Ginzburg-Landau (GL) equations are all mathematical descriptions of a fast oscillation modulated by a slow envelope. IN NLS and LL, the fast oscillation can be the crests of water waves, acoustic pressure oscillations or electric field oscillations. In the GL equation, the fast oscillation originates from a Turing or modulational instability. In either case, the problem is treated in a weakly nonlinear fashion, which can be systematically formalized by the mutliple scale method, and where the envelope is assumed to vary independently of the underlying oscillation. One thus ignore the fact that both slow and fast scale relate to one and the same space or time variable -and can therefore not be fully independent. A proper treatment of the interaction between the slow and fast scale in the weakly nonlinear limit involves exponentially small terms. Despite their smallness, these terms control the existence and position of fronts in the envelope, the interaction of distant pulses, and the phase of the fast oscillation relative to the slow envelope. The latter may become of importance in the limit of ultrashort pulses that contain only a few oscillations.

David LANNES

Laser filamentation and variants of the cubic NLS equation

The focusing cubic NLS is a canonical model for the propagation of laser beams. In dimensions 2 and 3, it is known that a large class of initial data leads to finite time blow-up. Now, physical experiments suggest that this blow-up does not always occur. This might be explained by the fact that some physical phenomena neglected by the standard NLS model become relevant at large intensities of the beam. Many ad hoc variants of the focusing NLS equation have been proposed to capture such effects. In this joint work with E. Dumas and J. Szeftel, we derive some of these variants from Maxwell's equations and propose some new ones. We also provide rigorous error estimates for all the models considered. Finally, we discuss some open problems related to these modified NLS equations and relate them to laser filamentation.

Luigi A. LUGIATO

The cavity Kerr medium model and its historical evolution

The talk starts from describing how the model (LL model [1]) was derived to provide a paradigm for optical pattern formation. The patterns which arise in 1D and 2D are easily identified with the guideline of nonlinear optics, which manifests also the quantum aspects of these phenomena. Cavity solitons in the spatial domain are discussed illustrating their control and their motional properties. Next, the discussion focusses on the temporal version of the LL model introduced by Haelterman, Trillo and Wabnitz and especially on temporal cavity solitons and their experimental observation. Finally the topic of broadband Kerr frequency combs is introduced.

[1] L. A. Lugiato and R. Lefever, *Phys. Rev. Lett. 58, 2209 (1987).*

Carles MILIAN

Soliton families and frequency combs in microring resonators: Higher-order dispersion and Raman effects

The influence of higher order dispersions and Raman scattering on solitons and frequency comb generation in microring resonators is investigated. Higher order dispersions sustain both conservative and dissipative solitons with large radiation tails that tend to have a stabilising effect. The Raman scattering introduces a threshold value in the resonator quality factor above which the frequency-locked solitons cannot exist, and instead, a rich dynamics characterised by the generation of self-frequency-shifting solitons is observed. The interplay of Raman effect, higher order dispersions, and Hopf bifurcations offers a mechanism for largely broadening the Cherenkov radiation spectrum of the frequency-locked solitons.

Tomoyuki MIYAJI

Bifurcation analysis of the Lugiato-Lefever equation in one space dimension

We consider a pattern-forming instability in a damped-driven nonlinear Schrödinger equation with a cubic nonlinearity called the Lugiato-Lefever equation. We study the bifurcation of a spatially homogeneous stationary solution for the equation and show that steady-state mode interactions occur. A numerical simulation suggests that the mode interaction plays an important role in forming localized patterns.

David NOVOA

Spontaneous symmetry breaking and shock-wave formation in gas-filled PCF ring cavities

We discuss the dynamics of a passive gas-filled PCF ring cavity synchronously pumped by short laser pulses. Depending on both duration and temporal profile of the circulating pulses, different phenomena are predicted. When pumping with ultrashort sech-shaped pulses, the system can exhibit spontaneous symmetry breaking in the pulse profiles, motivated by the non-trivial phase portrait acquired by the cavity pulses during their multiple interference events with the pump pulse train. In contrast, when the pump pulses are several picoseconds long and feature a super-Gaussian temporal structure, a novel kind of stable, stationary shock-wave is sustained by the ring configuration.

Wolfgang REICHEL

Uniqueness and bifurcation in the Lugiato-Lefever equation

In this talk I will show results on the behavior of bifurcating branches of solutions of the stationary Lugiato-Lefever equation under 2π-periodic boundary conditions. Moreover, based on recent a-priori estimates, I will show uniqueness of (trivial) solutions if the detuning parameter is large.

Frédéric ROUSSET

Long wave limits for Schrödinger maps

We will discuss the derivation of KdV type systems for some general geometric models (the Schrödinger maps) that can be seen as generalizations of the Gross-Pitaevskii equation. These models include the Landau Lifshitz equations (joint work with Pierre Germain (Courant Institute)).

Mustapha TLIDI

From localized structures to rogue waves in cavity nonlinear optics

We demonstrate a way to control rogue waves in a broad area nonlinear optical systems subjected to a delayed feedback. The delayed feedback is found to induce a spontaneous formation of rogue waves. In the absence of delayed feedback, spatial pulses are stationary. We characterize the formation of rogue waves by computing the probability distribution of the pulse height. The long-tailed statistical contribution which is often considered as a signature of the presence of rogue waves. The generality of our analysis suggests that the instability leading to the spontaneous formation of rogue waves in a controllable way is an universal phenomenon.
