Workshop on functional calculus and harmonic analysis of semigroups

Besançon, october 1-3, 2014

Abstracts of the talks

On a conjecture of Pisier on the analyticity of semigroups

Cédric Arhancet
Université de Franche-Comté

We show that the analyticity of semigroups $(T_t)_{t\geq 0}$ of selfadjoint contractive Fourier multipliers on L^p-spaces of compact abelian groups is preserved by the tensorisation of the identity operator of a Banach space for a class of K-convex Banach spaces, answering partially a conjecture of Pisier. We also give versions of this result for some semigroups of Schur multipliers and Fourier multipliers on noncommutative L^p-spaces.

Geometry and the Kato Square Root problem

Lashi Bandara
Chalmers University of Technology

The resolution of the Kato square root conjecture in 2002 by Auscher, Hofmann, Lacey, McIntosh and Tchamitchian was a momentous effort that brought together techniques from functional calculus and harmonic analysis. This problem, along with many other problems in harmonic analysis, was considered from a first-order perspective in the seminal work of Axelsson (Rosn), Keith and McIntosh in 2005. Here, the Kato Square Root problem on manifolds was considered for the first time and resolved in compact case. This was followed by the resolution of this problem on Euclidean submanifolds by Morris in 2010. We will present some recent results where this problem is resolved in the noncompact setting and where no embedding is assumed. Instead, we will show that the solution only makes assumptions on intrinsic curvature which signifies progress towards understanding the core geometric features of this problem. Furthermore, we will describe some geometric consequences, including applications to the study of manifolds with low-regularity metrics and possible connections to curvature flows.

From resolvent estimates to rates of decay

Charles Batty
University of Oxford

If the generator A of a bounded semigroup $\{T(t) : t \geq 0\}$ has no spectrum on the imaginary axis, then it follows that $\|T(t)A^{-1}\| \to 0$ as $t \to \infty$. Using functional calculus for the operator A and techniques from complex analysis, it is possible to give an almost precise rate of decay in terms of the rate of growth of $(is - A)^{-1}$. I will describe this result and its variants, the methods of proof, and its relevance to decay of energy for damped waves and to mixing properties in dynamical systems.

Functional calculus for generators of symmetric contraction semigroups

Andrea Carbonaro

Università degli Study di Genova

I shall show that every generator of a symmetric contraction semigroup on a σ-finite measure space admits, for $1 < p < \infty$, a Hörmander-type holomorphic functional calculus on L^p in the sector of angle $\phi_p^* = \arcsin|1 - 2/p|$. The obtained angle is optimal. This is a joint work with Oliver Dragičević.

Improved Sobolev Inequalities, semi-groups and stratified Lie groups

Diego Chamoro

Université d'Evry

We will see how the study of Sobolev inequalities in the setting of stratified Lie groups can be simplified by the use of semi-groups associated to a natural sub-Laplacian operator. Indeed, in this setting some of the most popular tools in analysis such as the Fourier transform can be successfully replaced by the properties of the heat semi-group associated to a functional calculus for the sub-Laplacian. We will obtain new results and also direct proofs for improved Sobolev inequalities using classical tools from harmonic analysis.

Structural Properties of Maximal Regularity

Stephan Fackler

University of Ulm

In this talk we study some structural aspects of maximal regularity. For example, a positive result by L. Weis says that the generator $-A$ of a positive contractive analytic C_0-semigroup on L^p for $p \in (1, \infty)$ has maximal regularity (and even a bounded H^∞-calculus). We present some known techniques to prove positive results and then motivate some open problems concerning the interactions between maximal regularity and properties of the semigroup such as positivity and contractivity. We conclude by giving partial negative answers to some of the questions.

Hardy spaces on graphs, application to Riesz transforms

Joseph Feneuil

Université Joseph Fourier

Let Γ be a graph with the doubling property and P a reversible random walk on Γ. We introduce Hardy spaces H^1 - for functions and for 1-forms - adapted to P and give various characterizations of it. In application, we state H^1-H^1 and H^1-L^1 boundedness of the Riesz transform.

Sobolev algebras through semigroup methods

Dorothee Frey
Université de Nantes

We consider Sobolev spaces L^p_α defined via an abstract self-adjoint operator generating a semigroup. A natural question to ask is whether $L^p_\alpha \cap L^\infty$ is an algebra under the pointwise product as in the Euclidean case. We shall give sufficient conditions on the semigroup to ensure such algebra properties, and describe implications for chain rules and paralinearisation formulas. Our results rely on abstract paraproducts and extrapolation methods in Lebesgue spaces. The results in particular apply to Sobolev spaces on doubling metric measure spaces endowed with a Dirichlet form.
This is joint work with F. Bernicot and T. Coulhon.

Square functions and functional calculus

Bernhard Haak
Université de Bordeaux

The talk discusses a joint paper with Markus Haase on square function estimates and H^∞-functional calculus. We show fairly simple principles that allow us to deduce "many" square functions from bounded H^∞ calculus and conversely the boundednesss of the calculus from some carefully selected square functions associated to semigroups, resolvents, etc.

Fundamental ideas in the theory of functional calculus and square functions

Markus Haase
Delft University of Technology

I will review some basic principles in the construction of functional calculi and present recent results about topological extensions. Then I will show how square functions emerge naturally as a certain extension of the notion of functional calculus, resulting in a calculus for square functions. Finally I plan to exemplify the power of this approach as well as shed some light on its limitations.
The talk is based on joint work with Panos Konstantopoulos (first part) and Bernhard Haak (second part).

The two-weight inequality for the Hilbert transform

Tuomas Hytönen
University of Helsinki

In two-weight estimates, the "input" and the "output" of an operator are weighted in unrelated ways, which is not unreasonable in applications. The problem of characterizing (in real-variable terms) the admissible pairs of weights for a two-weight inequality for the Hilbert transform was already raised by Muckenhoupt in the 1970s, but it was only recently solved by Lacey, Sawyer, Shen and Uriarte-Tuero in 2012-13. I extended their solution to general measures in place of weights, dealing in particular with the possibility of point masses. One of the key features of the proof is estimating certain tails of the Hilbert transform by comparison with the tails of the Poisson semigroup. An elaboration of this argument in the presence of point masses was the main additional challenge in my final result.

Hörmander functional calculus for non-selfadjoint operators

Christoph Kriegler

Université de Clermont-Ferrand

We present a method how to get a Hörmander functional calculus for an operator A acting on an $L^p(\Omega)$ scale with spectrum in $(0, \infty)$ out of kernel estimates for the semigroup $\exp(-zA)$ and an H^∞ calculus for A on $L^2(\Omega)$. Here Ω is a space of homogeneous type with polynomial volume growth of some order $d \geq 1$ and with a certain growth bound for volumes of annuli, and the kernel of the semigroup is assumed to be bounded by the Poisson kernel in dimension d, for complex times $z \in \mathbb{C}_+$. The novelties are that the Poisson kernel decays slower than the Gaussian kernel which is usually assumed, and that only a bounded H^∞ calculus is needed in place of self-adjointness of A. We apply the result to a (non-selfadjoint) example of a Lamé operator acting on $L^p(\mathbb{R}^d; \mathbb{C}^{d+1})$.

Generalized Triebel-Lizorkin spaces for differential operators

Peer Kunstmann

Universität Karlsruhe

For an R_q-sectorial operator A in Banach function space X we construct intermediate spaces between X and the domain X_1 of the operator which generalize for $X = L^p(\mathbb{R}^n)$ and $A = -\Delta$ the Triebel-Lizorkin spaces $F^{2\theta}_{p,q}(\mathbb{R}^n)$. We give applications to the H^∞-functional calculus for some classes of differential operators.
This is joint work with Alexander Ullmann.

Poisson semigroups and Riesz transform inequalities

Tao Mei

Wayne State University

Let $T_t = e^{-tL}, 0 \leq t < \infty$ be a Markov semigroup of operators on $L^p(M)(1 \leq p \leq \infty)$. P. A. Meyer suggests an abstract gradient form ∇_L associated with its generator and asks whether the following associated Riesz transform inequality holds,

$$\|\nabla_L f\|_p \simeq \|L^{\frac{1}{2}} f\|_p \tag{1}$$

D. Bakry proved that P. A. Meyer's inequality (**??**) holds for any *diffusion* Makrov semigroup satisfying a Γ_2 condition. By *diffusion*, they mean that the semigroups admits a Markov dilation with almost continuous path. The most important non-diffusion Markov semigroup maybe the classical Poisson semigroups $P_t = e^{-t\sqrt{-\Delta}}$ on $\mathbb{R}^n$. What we can say on a possible Riesz transform inequality associated with P_t?
This talk will report recent joint work with M. Junge and J. Parcet on P. A. Meyer's inequality for Markov semigroups on $L^p(M)$ in the case that M is a "dual group" (e.g. Pontryagin dual of a locally compact abelian group). In this case, P. A. Meyer's inequality (**??**) holds for all Markov semigroups with a necessary "noncommutative" modification.

Spectral multipliers and restriction estimates via the derivatives of the corresponding semigroup

El Maati Ouhabaz

Université de Bordeaux

We report on recent results on sharp spectral multipliers for self-adjoint operators L. Given a bounded funtion F on $\mathbb{R}^+$, the operator $F(L)$ is bounded on L^2 by the standard functional calculus. The basic question is to find optimal conditions on F which allow to extend $F(L)$ to a bounded operator on L^p for p in some interval around 2. The conditions we have in mind are in the spirit of Mikhlin or Hörmander's Fourier multiplier theorem for the Euclidean Laplacian. We introduce restriction estimates for general operators L which imply sharp spectral multiplier results. The restriction estimates can be viewed as $L^p - L^{p'}$-boundedness of the spectral measure of L. We show how precise estimates on the derivatives of the semigroup of L imply these restriction estimates.

This talk is based on the papers:

- P. Chen, E.M. Ouhabaz, A. Sikora and L. Yan: Restriction estimates, sharp spectral multipliers and endpoint estimates for Bochner-Riesz means (J. Analyse Mathématique).
- F. Bernicot and E.M. Ouhabaz: Restriction estimates via the derivatives of the heat semigroup and connection with dispersive estimates (Math. Res. Letters).

Functional calculus of Dirac operators and tent spaces

Pierre Portal

Australian national University and Université de Lille

In [2], Axelsson, Keith, and McIntosh have shown that results about Riesz transforms, such as the boundedness of the Cauchy integral on Lipschitz curves, and Kato's square root estimates, can be seen as instances of a perturbation result, in L^2, for the H^∞ functional calculus of certain first order differential operators. This perspective has since been proven to be particularly useful for the development of harmonic analysis on manifolds, and in the study of rough boundary value problems. It has been extended from L^2 to L^p in two different ways: using an appropriate extrapolation method in [1], or a set of martingale techniques that provide L^p analogues of the key techniques of [2] in [3].

In this talk, we present an alternative set of L^p techniques based on Hardy spaces and heat kernel bounds rather than martingales and R-boundedness methods. This turns out to be simpler and give stronger results, and also points out an interesting phenomenon: the heart of the harmonic analysis in [2] actually extends from L^2 to L^p for all $p \in (1, \infty)$, while the (necessary) restrictions in p only come from an operator theoretic estimate that is trivial in L^2. Our approach is fundamentally based on Coifman-Meyer-Stein's theory of tent spaces, and on the current development of an operator-valued Calderón-Zygmund theory in these spaces.

This is joint work with D. Frey and A. McIntosh.

[1] P. Auscher, On necessary and sufficient conditions for L^p-estimates of Riesz transforms associated to elliptic operators on $\mathbb{R}^n$ and related estimates. *Mem. Amer. Math. Soc. 871* (2007).

[2] A. Axelsson, S. Keith, A. McIntosh, Quadratic estimates and functional calculi of perturbed Dirac operators. *Invent. Math. 163* (2006) 455–497.

[3] T. Hytönen, A. McIntosh, P. Portal, Kato's square root problem in Banach spaces. *J. Funct. Anal. 254* (2008) 675–726.

Hypercontractivity and free products

Eric Ricard
Université de Caen

We will discuss hypercontractivity in the setting of noncommutative Lp-space. We focus on semigroups arising from the free product construction.

Conical square functions associated with Bessel, Laguerre and Schrödinger operators in UMD Banach spaces

Lourdes Rodríguez Mesa
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Universidad de La Laguna

We show new equivalent norms for functions in the Lebesgue-Bochner spaces $L^p(\Omega, \mathbb{B})$, $1 < p < \infty$, $\Omega \subset \mathbb{R}^n$ and $\mathbb{B}$ a UMD Banach space. These norms are given in terms of conical square functions associated to the Bessel, to the Laguerre and to the Schrödinger operators and defined by using γ-radonifying operators.
The results that will be exposed in the talk were obtained in collaboration with Jorge J. Betancor, Alejandro J. Castro and Juan Carlos Fariña (Universidad de La Laguna).

Functional calculus on interpolation spaces for groups

Jan Rozendaal
Delft University of Technology

In this talk some recent results on functional calculus for generators of strongly continuous groups on real interpolation spaces will be explained. Interpolation versions of known transference principles for groups are established that link harmonic analysis to functional calculus theory. In particular, let $(U(s))_{s\in\mathbb{R}}$ be a bounded strongly continuous group with uniform bound $M \in [1, \infty)$ on a Banach space X. The classical transference principle of Berkson, Gillespie and Muhly [1] yields an estimate

$$\left\| \int_{\mathbb{R}} U(s)x\,\mu(\mathrm{d}s) \right\|_X \leq M^2 \, \|L_\mu\|_{\mathcal{L}(\mathrm{L}^p(X))} \, \|x\|_X$$

for all $x \in X$ and each complex Borel measure μ on $\mathbb{R}$, where L_μ is convolution with μ on $\mathrm{L}^p(X)$, the space of p-integrable X-valued functions for $p \in [1, \infty]$. We describe how to deduce a version of this transference principle on real interpolation spaces between X and the domain of the generator A of $(U(s))_{s\in\mathbb{R}}$. This then allows one to use results about Fourier multipliers on vector-valued Besov spaces to obtain statements on the boundedness of certain functional calculi for group generators on real interpolation spaces. These results do not depend on the geometry of the underlying Banach space, in particular they hold on spaces which do not have the UMD property. From this in turn results about principal value integrals, sectorial operators and generators of cosine functions can be deduced.
This talk is based on joint work with Markus Haase.

[1] Earl Berkson, T. Alistair Gillespie, and Paul S. Muhly. Generalized analyticity in UMD spaces. *Ark. Mat.*, 27(1):1-14, 1989.

ON SUBORDINATION THEORY OF HOLOMORPHIC SEMIGROUPS

Yuri Tomilov

Institute of Mathematics, Polish Academy of Sciences, Warsaw

We present solutions of several open problems on subordination of holomorphic semigroups.
First, we prove that that for any Bernstein function ψ the operator $-\psi(A)$ generates a holomorphic C_0-semigroup $(e^{-t\psi(A)})_{t\geq 0}$ on a Banach space, whenever $-A$ does. This answers a question posed by Kishimoto and Robinson in 1981. Second, giving a positive answer to a question by Berg, Boyadzhiev and de Laubenfels from 1993, we show that $(e^{-t\psi(A)})_{t\geq 0}$ is holomorphic in the holomorphy sector of $(e^{-tA})_{t\geq 0}$, and if $(e^{-tA})_{t\geq 0}$ is sectorially bounded in this sector then $(e^{-t\psi(A)})_{t\geq 0}$ has the same property.
If time permits, we also present new sufficient conditions on ψ in order that, for every Banach space X, the semigroup $(e^{-t\psi(A)})_{t\geq 0}$ on X is holomorphic whenever $(e^{-tA})_{t\geq 0}$ is a bounded C_0-semigroup on X. These conditions improve and generalize well-known results by Carasso-Kato and Fujita (1991-93).
This is joint work with A. Gomilko (Torun).

R-BOUNDEDNESS VERSUS γ-BOUNDEDNESS

Lutz Weis

Universität Karlsruhe

The notions of R- bounded and γ bounded operator families generalizes classical square function estimates in harmonic analysis. Therefore they have found many applications to Fourier multiplier theorems, the holomorphic functional calculus and spectral multiplier theorems. It is well-known that in Banach spaces with finite cotype, the R-bounded and γ-bounded families of operators coincide. If in addition X is a Banach lattice, then these notions can be expressed as square function estimates. It is also clear that R-boundedness implies γ-boundedness. In this talk we report on some joint work with S. Kwapien and M. Veraar showing that all other possible inclusions fail. Furthermore, we will prove that R-boundedness is stable under taking adjoints if and only if the underlying space is K-convex