Workshop on Theoretical and Numerical Topics in Control and Inverse Problems for PDEs
Abstracts

Laboratoire de Mathématiques de Besançon (France)

March 02-07, 2015

Contents

1 Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
$8\frac{1}{2}$-$9\frac{1}{2}$		Benabdallah	Boyer	Cristofol	Laurent	Fernand.-Cara
$9\frac{1}{2}$-$10\frac{1}{2}$	Welcome	Boyer	Benabdallah	Cîndea	Morancey	Lissy
	Coffee Break					
11-12	Boyer	Alabau	Tucsnak	Glass	Komornik	Gonz.-Burgos
12-13	Benabdallah	Maniar	de Teresa	Guerrero	Le Gorrec	
	Lunch					
$14\frac{1}{2}$-$15\frac{1}{2}$	Ammari	Rao		Münch	Fernand.-Cara	
$15\frac{1}{2}$-$16\frac{1}{2}$	Pandolfi	Teniou		Valein	Le Rousseau	
	Coffee Break			**Coffee Break**		
17-18	Dermenjian	Hubert		Bouzidi	Bennour	
18-$18\frac{1}{2}$	Queiroz	A. Souza				

2 Fatiha Alabau-Boussouira: Indirect control and observability of cascade systems of PDE's

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We present an overview of the questions, and recent positive and negative results on control and observability for coupled hyperbolic cascade systems. We are interested in situations for which the number of controls (or observations) is strictly less than the number of unknowns (or of equations), the controls (or observations) being limited to certain components of the state-vector. This is called indirect control (or observability). The challenging questions are then, to determine whether it is possible to drive back the full system, that is all the components of the state-vector, to a desired state in finite time. We present positive results as well as negative results, and in particular stress the influence of the coupling operators on explicit and constructive examples.

3 Kaïs Ammari: Local feedback stabilization to a non-stationary solution for a damped non-linear wave equation

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We study a damped semi-linear wave equation in a bounded domain of $\mathbb{R}^3$ with smooth boundary. It is proved that any H^2-smooth solution can be stabilized locally by a finite-dimensional feedback control supported by a given open subset satisfying a geometric condition.

The proof is based on an investigation of the linearised equation, for which we construct a stabilizing control satisfying the required properties. We next prove that the same control stabilizes locally the non-linear problem. This is joint work with Thomas Duyckaerts and Armen Shirikyan.

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4 Assia Benabdallah: Survey on controllability of Linear Parabolic Systems

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The aim of these lectures is to give an overview on the controllability of parabolic systems.

In the first lecture, I will consider internal controllability of parabolic equations coupled by constant coefficients. I will show that, as in the finite dimensional case, the controllability for this class of systems reduces to a Kalman type condition.

In the second lecture, I will consider the boudary controllability of the same class of systems. Through simple examples, I will show that new phenomena appear.

In the third lecture, I will return to the internal controllability considered in the first lecture, but for parabolic equations coupled by space dependent coefficients. Here also, through simple examples, I will show that new phenomena appear.

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5 Abdelaziz Bennour: Control of coupled hyperbolic systems via the method of moments

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In this talk, we consider the following system:

$$\begin{cases} y_{tt} = y_{xx} + a\left(x\right) z_t & \text{dans } Q = (0,\pi) \times (0,T) \\ z_{tt} = \delta^2 z_{xx} & \text{dans } Q = (0,\pi) \times (0,T) \\ y(0,t) = v(t) \ \ , \ \ y(\pi,t) = 0 & t \in (0,T) \\ z(0,t) = v(t) \ \ , \ \ z(\pi,t) = 0 & t \in (0,T) \\ y(x,0) = y^0 \ , \ y_t(x,0) = y^1 & \text{dans } \Omega \\ z(x,0) = z^0 \ , \ z_t(x,0) = z^1 & \text{dans } \Omega \end{cases} \tag{1}$$

where $\delta \neq 0$ is a real constant, $a \in \mathcal{C}^1\left([0,\pi]\right)$ and $v \in L^2\left(0,T\right)$. We study the exact controllability of this system via the methof of moments.

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6 Franck Boyer: Autour de la méthode HUM et de ses applications pour le calcul numérique de contrôles à zéro d'EDPs d'évolution

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Dans ce mini-cours je commencerai par discuter quelques propriétés simples de la méthode HUM (Hilbert Uniqueness Method) en particulier dans sa version pénalisée. Le point de vue adopté, relativement général et abstrait, a pour but de mieux appréhender l'application de cette approche utilisée depuis plus de 20 ans pour le calcul numérique de contrôles pour des problèmes paraboliques. Il s'agit par exemple d'étudier comment les notions usuelles de contrôlabilité ou d'observabilité peuvent se comprendre dans un contexte où on considère une famille de problèmes de contrôle dépendant d'un paramètre (typiquement celui de la discrétisation).

On s'intéressera ensuite à l'analyse de situations concrètes pour lesquelles on peut prouver des propriétés de "contrôlabilité uniforme" par rapport aux paramètres de discrétisation en espace et/ou en temps de tels systèmes. Il sera ainsi question par exemple d'inégalités de Carleman discrètes et de leurs applications.

L'ensemble sera illustré par différents exemples numériques 1D et 2D à la fois dans des situations bien comprises sur le plan théorique que dans des cas où l'analyse est encore largement ouverte.

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7 Chérif Bouzidi: Null controllabilty of retarded parabolic equations

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We adress in this work the null controllability problem for linear heat equation with delay parameters. The control is exerted on a subdomain and we show how the global Carleman estimate due to Fursikov and Imanuvilov can be applied to derive these results in this direction.

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8 Nicolae Cïndea: Observateurs sous-échantillonnés en temps pour l'équation des ondes

Le but de cet exposé est de proposer des schémas d'assimilation des données, en utilisant des observateurs de Luenberger, dans des situations quand seulement des données sous-échantillonnées en temps sont disponibles. Plus précisément nous considérons l'équation des ondes pour laquelle nous supposons connue la restriction de sa solution à un sous-ensemble de son domaine en espace et pour certains temps. Pour assimiler les données disponible nous proposons deux stratégies : la première consiste à assimiler les données quand elles sont disponible et la deuxième consiste à assimiler à tout temps les données obtenues en interpolant en temps les observations disponibles. Pour les deux stratégies on démontre des estimations des erreurs entre la solution exacte et les observateurs proposés. Des simulations numériques illustrent les résultats théoriques.

Ce travail a été réalisé en collaboration avec Alexandre Impériale et Philippe Moireau.

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9 Michel Cristofol: Inverse boundary value problem for the dynamical heterogeneous Maxwell's system

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We consider the inverse problem of determining the isotropic inhomogeneous electromagnetic coefficients of the non-stationary Maxwell equations in a bounded domain of $\mathbb{R}^3$, from a finite number of boundary measurements. The main result is a Holder stability estimate for the inverse problem, where the measurements are exerted only in some boundary components. For it, we prove a global Carleman estimate for the heterogeneous Maxwell's system with boundary conditions. Then we exhibit some numerical simulations.

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10 Yves Dermenjian: Carleman estimates for discontinuous media: the case of an elliptic operator with discontinuous coefficients through an interface transverse to the boundary

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We consider the following situation

- $\Omega = \Omega' \times (-1,1) \subset \mathbb{R}^{n-1} \times \mathbb{R}, \Omega, \Omega'$ are bounded open sets,
- $A = -\nabla \cdot B\nabla$ is an elliptic operator with

$$B(x) = B(x', x_n) = \begin{pmatrix} c_1(x)C_\tau(x') & 0 \\ 0 & c_2(x) \end{pmatrix},$$

  where c_1, c_2 are scalar functions and C_τ is a $(n-1)\times(n-1)$ matrix,
- the coefficients of C_τ belong to $C^1(\Omega')$,
- $c_1, c_2 \in C^1(\overline{\Omega_\pm})$ with a possible jump at the interface $\{x_n = 0\}$ $(\Omega^+ = \Omega'^- = \Omega' \times (-1,0))$.

We shall detail a method that leads to a Carleman estimate with a distributed control (Dirichlet condition on the boundary) not using a semiclassical approach. If we set $Au = f$, the main difficulty is the value of the derivative of u along the interface. We shall point out some limits of our approach and challenges for the future.

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11 Enrique Fernández Cara: Controllability, turbulence, hierarchical control and free-boundaries

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I will review some recent results dealing with the control of PDEs that provide acceptable answers. They have been obtained in collaboration with F. Araruna, J. Limaco, S.B. Menezes, A. Münch, M.C. Santos and D.A. Souza. Specifically, we will be concerned with

1. Controllability results for the Leray-α model of turbulence.
2. Theoretical and numerical control of the Ladyzhenskaja-Smagorinsky model.
3. Hierarchical control, Stackelberg-Nash strategies and applications.
4. Controllability of free-boundary problems and related fields.

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12 Olivier Glass: A controllability result for the the non-isentropic 1-D Euler equation

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We examine the question of the boundary controllability of the one-dimensional non-isentropic Euler equation for compressible polytropic gas, in the context of weak entropy solutions. We consider the system in Eulerian coordinates and the one in Lagrangian coordinates. For both systems a result of controllability toward constant states is obtained (with a limitation on the adiabatic constant for the Lagrangian system). Moreover the solutions that are constructed remain of small total variation in space for all time.

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13 Manuel González-Burgos: Minimal time of controllability of two parabolic equations with disjoint control and coupling domains

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In this talk we will analyze the controllability properties at time $T > 0$ of a system of two parabolic equations coupled by a matrix $A(x) = q(x)A_0$, where A_0 is a Jordan block of order 1, when we exert one distributed or boundary control. The support of the coupling coefficient, q, and the control domain may be disjoint. We will show that, in general, there exists an explicit minimal time $T_0(q) \in [0, \infty]$ such that if $T > T_0(q)$, then the system is null-controllable at time T and if $T < T_0(q)$ the null controllability of the system at time T fails. We will also see that this minimal time depends on q and on the geometrical positions of the control open and the support of q.

14 Sergio Guerrero: On the controllability of micropolar fluids

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15 Florence Hubert: Modélisation de la croissance tumorale et métastatique. Un nouveau terrain de jeu pour les contrôleurs.

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Dans cet exposé nous décrirons quelques modèles mathématiques de croissance tumorale et métastatique et de traitement anti-cancéreux. Nous montrerons comment les problématiques des médecins peuvent s'exprimer en terme de problème de contrôle ou de contrôle optimal.

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16 Vimos Komornik: Ingham type theorems

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We present older and some recent results on the application of nonharmonic Fourier series in control theory, obtained in collaboration with C. Baiocchi, P. Loreti and B. Miara.

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17 Camille Laurent: Bilinear control of Schrödinger equations

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In this talk, we will discuss about the controllability of the Schrödinger equation when the control act as a potential term in the equation. We will discuss two cases:

- the $1D$ case where we only control the amplitude of a potential with a fixed profile. The proof will make use of a "regularizing effect".
- the 2D case where the potential satisfies a Poisson equation and the control is the boundary value of the potential. We will make the link with the more standard boundary control of the Schrödinger equation.

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18 Yann Le Gorrec: Energy shaping of boundary controlled port Hamiltonian systems

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In this talk we present a general methodology for the synthesis of asymptotic or exponential stabilizing boundary control laws for a large class of linear distributed parameters systems defined on a one-dimensional spatial domain. We first present the control design by immersion/reduction approach. In this case the boundary controlled distributed parameter system is first connected to a dynamic controller. The closed loop stystem is then reduced trough the use of its structural invariants, the so called Casimir functions. This approach is of great interest as it allows to shape the closed loop energy function and leads to an asymptotically stabilizing controller. Unfortunately this methodology fails in presence of strong dissipation as the closed loop structural invariants are broken. In the second part of this talk we show that in this case we can use the same ideas and state feedback (that is the price to pay) to shape the closed loop energy function and to asymptotically or exponentially stabilize the system. This approach will be related to backstepping approaches.

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19 Jérôme Le Rousseau: Null-controllability of the Kolmogorov equation in the whole phase-space

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We shall present how existing results on the null controllability in two-dimensional phase-space for the Kolmogorov equation can be extended to arbitrary even dimensions, and to fairly general control-region geometries. This is joint work with Ivan Moyano (Ecole Polytechnique).

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20 Pierre Lissy: Uniform controllability of the one-dimensional convection-diffusion equation and cost of fast controls for the heat equation

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In this talk, I will present some results concerning the minimal time needed to ensure the uniform controllability of the convection-diffusion equation with positive or negative speed. I will also explain the link between some conjectures on this problem and the usual conjectures concerning the cost of observation/control of the heat equation.

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21 Lahcen Maniar: Null controllability for parabolic equations with dynamic boundary conditions

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We prove null controllability for linear and semilinear heat equations with dynamic boundary conditions of surface diffusion type. The results are based on a new Carleman estimate for this type of boundary conditions.

22 Morgan Morancey: Minimal time for null controllability of degenerate parabolic equations: the case of Grushin-type equations

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In this talk I will present some recent results concerning null controllability of degenerate parabolic equations. I will explain the technics and strategies used in previous works leading to the possible existence of a minimal time for null controllability. I will then focus on a recent collaboration with K. Beauchard and L. Miller where we caracterized and computed the this minimal time in certain geometric configurations. The proof uses a transmutation strategy and detailed energy estimates on the resulting wave equations.

23 Arnaud Münch: Inverse problems for linear hyperbolic equations via mixed formulations

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We explore a direct method allowing to solve numerically inverse type problems for hyperbolic type equations. We first consider the reconstruction of the full solution of the wave equation posed in $\Omega \times (0,T)$ - Ω a bounded subset of $\mathbb{R}^N$ - from a partial distributed observation. We employ a least-squares technic and minimize the L^2-norm of the distance from the observation to any solution. Taking the hyperbolic equation as the main constraint of the problem, the optimality conditions are reduced to a mixed formulation involving both the state to reconstruct and a Lagrange multiplier. Under usual geometric optic conditions, we show the well-posedness of this mixed formulation (in particular the inf-sup condition) and then introduce a numerical approximation based on space-time finite elements discretization. We show the strong convergence of the approximation and then discussed several examples for $N = 1$ and $N = 2$. The reconstruction of both the state and the source term is also discussed, as well as the boundary case as well as the parabolic case. Joint works with Nicolae Cîndea. Details can be found in [1] using arguments developed in [2, 3].

References

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[3] N. Cîndea and A. Münch, *A mixed formulation for the direct approximation of the control of minimal L^2-norm for linear type wave equations*, Calcolo, 2015.

24 Luciano Pandolfi: Control problems for systems with persistent memory

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We survey recent results on controllability of distributed systems with persistent memory under the action of a control acting in the Dirichlet boundary condition. In concrete applications to thermodynamics or viscoelasticity, this amounts to impose a temperature to (a part of the) boundary of a thermal system with memory, or a boundary deformation to a viscoelastic body.

We concentrate mainly on the boundary control of a system of Gurtin-Pipkin type, i.e. a system described by

$$w'' = \Delta w + \int_0^t M(t-s)\Delta w(s)\mathrm{d}\, s\,. \tag{2}$$

Here $w = w(x,t)$ where $x \in \Omega \subseteq \mathbb{R}^n$ (a region with smooth boundary). We impose

$$w(x,0) = 0\,, \quad w'(x,0) = 0\,, \qquad w = f \text{ on } \Gamma \subseteq \partial\Omega\,.$$

The *associate wave equation to (2)* is the equation (2) with $M = 0$.

Like in the case of the wave equation, controllability is the property that it is possible to hit a target $(\xi, \eta) \in L^2(\Omega) \times H^{-1}(\Omega)$ by using a suitable (square integrable) control. Note that, differently from the wave equation, this property is not controllability in the sense of Kalman, but it extends the property of “relative controllability” in the sense of Gabasov and Kirillova. It turns out that this (weaker) controllability property is the right concept for the identification of initial conditions (when they are nonzero, i.e. observability) and source reconstruction, see [4].

It has been proved in recent times that the controllability properties of the associated wave equation are inherited by the equation (2) with the same “sharp” control time, regardless of the (smooth) kernel $M(t)$. We show an efficient proof which combines the moment methods developed in [5, 6] with the cosine operator approach (in [3] and [6, Chp. 2]).

We contrast the controllability properties of (2) with recent results (in [1, 2]) on controllability of Colemann-Gurtin type systems

$$w' = \Delta w + \int_0^t N(t-s)\Delta w(s)\mathrm{d}\, s\,. \tag{\textbf{B}}$$

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25 Pammella Queiroz de Souza: Asymptotic limits of the Timoshenko system and stability

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When, in 1988, Lagnese-Lions proved that the asymptotic limit of the Timoshenko system converges to Kirchhoff system, many other questions about the asymptotic limit have appeared. Among them, we can cite: What happens with the limit of Timoshenko's system if we consider the modulus of elasticity of the shear fixed and that the thickness of the beam tends to zero? The aim of this conference is to give an answer for this and other questions about asymptotic limit for the Timoshenko system by generalizing some results (due to Lagnese and Lions on the one hand, and to Araruna and Zuazua on the other). This is a joint work with F. Ammar-khodja.

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26 Bopeng Rao: Exact synchronization for a coupled system of wave equations with Dirichlet boundary controls

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In this paper the exact synchronization for a coupled system of wave equations with Dirichlet boundary controls and some related concepts are introduced. By means of the exact null controllability of a reduced coupled system, under certain conditions of compatibility, the exact synchronization, the exact synchronization by groups are all realized by suitable boundary controls. The final state he exact synchronization will be also discussed.

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27 Diego A. Souza: On the numerical controllability problem for the heat, Stokes and Navier-Stokes equations

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This talk deals with the numerical computation of distributed null controls for the heat and Stokes equations and exact to the trajectory controls for the Navier-Stokes equations. The main idea is to minimize over the class of admissible null controls a functional that involves weighted integrals of the state and the control, with weights that blow up near the nal time. The associated optimality conditions can be viewed as a differential system in the variables $(x;t)$ that is second order in time and fourth order in space, completed with appropriate boundary conditions. We present several mixed formulations of the problems and, then, appropriate mixed nite element approximations that rely on Lagrangian C^0 spaces.

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28 Djamel Teniou: Carleman estimates for Ventcel boundary conditions of parabolic equations and applications

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We consider the heat equation in a domain $\Omega := (0, L) \times \Omega' \subset \mathbb{R} \times \mathbb{R}^{n-1}$, with Dirichlet conditions on a part $\Gamma \subset \partial\Omega$ of the boundary, and Ventcel conditions on $\partial\Omega\backslash\Gamma$. This condition is itself a PDE of ordre two.

We give a Carleman estimates which contains terms of boundary.

As application, we present some results concerning the controlability of this system.

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29 Luz de Teresa: On hierarchic control for coupled parabolic equations

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In this conference we will discuss the problem of hierarchic control for two coupled parabolic equations. We start recalling the problem for a single heat equation and will see what happens as soon as two coupled equations are considered. We present recent results and propose open problems.

This work is in collaboration with Victor Hernández-Santamaría.

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30 Marius Tucsnak: Analysis and control of the piston problem

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We consider two systems modelling the motion of a piston in a compressible fluid. The first model couples the equations of a viscous heat conducting gas with the ODE modelling the equation of a piston. We first recall classical results of Shelukhin, providing a global existence result For the same model we discuss the large time behaviour, possibly in the presence of a feedback force acting on the piston. In the main part of the talk we consider a simplified model, in which the equations of nonisothermal gas dynamics are replaced by the viscous Burgers equation. In this case we provide global exact/approximate controllability results in large time.

31 Julie Valein: Observers for populations dynamics

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We consider the McKendrick-Von Foester system with diffusion

$$\begin{cases} \partial_t p(a,x,t) + \partial_a p(a,x,t) = -\mu(a)p(a,x,t) + k\Delta p(a,x,t), & a \in (0,a^*),\ x \in \Omega,\ t > 0, \\ p(a,x,t) = 0, & a \in (0,a^*),\ x \in \partial\Omega,\ t > 0, \\ p(a,x,0) = p_0(a,x), & a \in (0,a^*),\ x \in \Omega, \\ p(0,x,t) = \displaystyle\int_0^{a^*} \beta(a)p(a,t,x)\,\mathrm{d}a, & x \in \Omega,\ t > 0, \end{cases} \tag{3}$$

where

- $p(a,x,t)$ denotes the distribution density of the population of age a at spatial position x at time t;
- p_0 denotes the initial distribution;
- a^* is the maximal life expectancy;
- $\beta(a)$ and $\mu(a)$ are positive functions denoting respectively the birth and death rates (which are supposed to be independent of x);
- Ω denotes a smooth bounded domain, k is a positive constant diffusion coefficient and Δ the laplacian with respect to the space variable x.

The aim of this talk is to contruct an observer for (3) in order to recover the distribution density of the population $p(a, x, T)$ at any age a, at time T large enough and at any spatial position x, knowing the distribution density of the population $p(a, x, t)$ at any time t but only on a small region $\mathcal{O} \subset \Omega$ and on an age interval (a_1, a_2) (where a_1, $a_2 \in (0, a^*)$), i.e. knowing $y(t) = p|_{(a_1,a_2)\times\mathcal{O}}$ where $t \in (0, T)$ (and assuming that p_0 is unknown).

The construction of the observer is based on the spectral properties of the population operator (see [2]) and on a splitting in a finite dimensional system to be stabilized and an infinite dimensional stable system (see [1] for instance).

This is joint work with Karim Ramdani and Marius Tucsnak.

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